نام و نام خانوادگی: آیلین عظیمی — تکلیف ۷ — کلاس: [illegible]

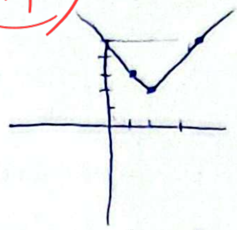
۱۹٫۷۵

۱

الف) $y = x + |2x-4|$:

$x<2$	$x\ge 2$
$x-2x+4 = -x+4$	$x+2x-4 = 3x-4$

$f(1)=3 \quad f(2)=2 \quad f(3)=5$



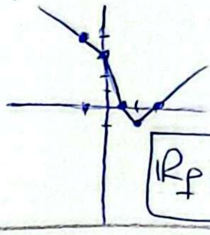
$R_f:[2,+\infty)$

(۲)

ب) $y=|x-2|+|x-1|-|x|$:

$x<0$	$0\le x<1$	$1\le x<2$	$x\ge 2$
$-x+2-x+1+x = -x+3$	$-x+2-x+1-x = -3x+3$	$-x+2+x-1-x = -x+1$	$x-2+x-1-x = x-3$

$f(-1)=4 \quad f(0)=3 \quad f(1)=0 \quad f(2)=-1 \quad f(3)=0$



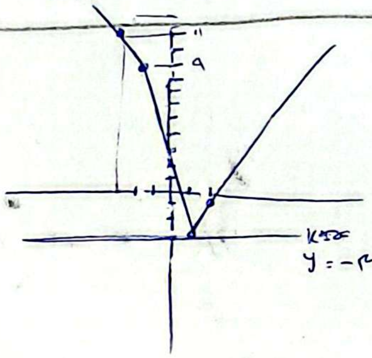
$R_f:[-1,+\infty)$

۲

$|3x-3|-|x+2| = k$

$x<-2$	$-2\le x<1$	$x\ge 1$
$-3x+3+x+2 = -2x+5$	$-3x+3-x-2 = -4x+1$	$3x-3-x-2 = 2x-5$

$f(-3)=11 \quad f(-2)=9 \quad f(1)=-3 \quad f(2)=-1$



تنها خطی که در اثر برخورد با شکل ۱ ریشه می‌دهد ← فقط $y=-3$ است

$\boxed{k=-3}$

(۲)

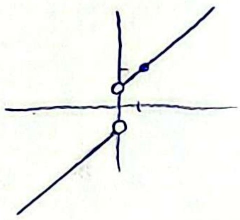
۳

الف) $y = x+\frac{x}{|x|}$:

$x<0$	$x>0$
$x-1$	$x+1$

$f(-1)=-2 \quad f(0)=X \quad f(1)=2$

$x\neq 0$

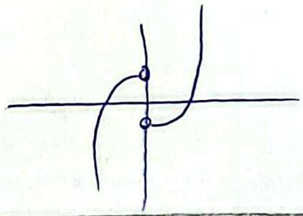


(۲)

ب) $y = x|x| - \frac{x}{|x|}$:

$x<0$	$x>0$
$-x^2+1$	x^2-1

$x\neq 0$



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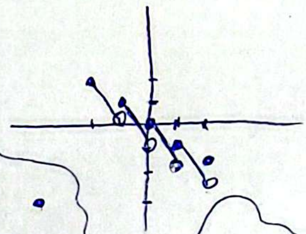
الف) $y=[x]-2x \quad \xrightarrow{-2\le x\le 2}$



$-2\le x<-1 \xrightarrow{[x]=-2} y=-2x-2$

$-1\le x<0 \xrightarrow{[x]=-1} y=-2x-1$

$0\le x<1 \xrightarrow{[x]=0} y=-2x$

$1\le x<2 \xrightarrow{[x]=1} y=-2x+1$

$x=2 \longrightarrow [x]=2 : y=-2x+2$

(۲)

ب) $y=[x]|x| \quad \xrightarrow{-2\le x\le 2}$

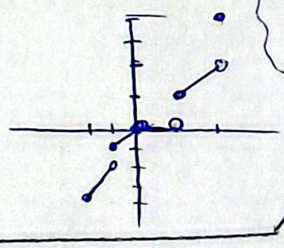
$-2\le x<-1 : (-2)(-x)=2x$

$-1\le x<0 : (-1)(-x)=x$

$0\le x<1 : (0)(x)=0$

$1\le x<2 : (1)(x)=x$

$x=2 : (2)(x)=2x$



ج) $y=x-5\left[\frac{x}{5}\right] = 5\left(\underbrace{\frac{x}{5}-\left[\frac{x}{5}\right]}_{[0,1)}\right) \to \boxed{R_f:[0,5)}$

د) $y=[2x]-2x \to 0\le 2x-[2x]<1$

$0\ge [2x]-2x>-1 \to \boxed{R_f:(-1,0]}$

۵

الف) $y=3x-[3x] \to 0\le \square-[\square]<1 \to \boxed{R_f:[0,1)}$

ب) $y=4x-4[x] = 4(x-[x]) \to 0\le x-[x]<1$

$0\le 4(x-[x])<4 \to \boxed{R_f:[0,4)}$

(۲)

الف) $y = 3\sin x + 2\cos x$ $\quad [-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$ $\quad -\sqrt{13} \le 3\sin x + 2\cos x \le \sqrt{9+4} = \sqrt{13} \Rightarrow \boxed{R_f : [-\sqrt{13}, \sqrt{13}]}$

ب) $y = 4\sin x - 3\cos x \rightarrow -5 \le 4\sin x - 3\cos x \le \sqrt{16+9} = 5 \Rightarrow \boxed{R_f : [-5, 5]}$ 5 6

ج) $y = [2\sin x + 5\cos x] \rightarrow -\sqrt{29} \le 2\sin x + 5\cos x \le \sqrt{4+25} \Rightarrow [-\sqrt{29}, \sqrt{29}] : \boxed{R_f : \{y \mid y \in Z, -6 \le y \le 5\}}$

د) $y = \sqrt{\cos x - 2\sin x} \rightarrow -\sqrt{5} \le \cos x - 2\sin x \le \sqrt{1+4} \rightarrow \sqrt{[-\sqrt{5}, \sqrt{5}]} \Rightarrow \boxed{R_f : [0, \sqrt[4]{5}]}$

$-\frac{b}{2a} = -\frac{3}{2} \notin$

Max $\sin x : -1 = (0)(1) = 0$

الف) $y = \sin^2 x + 3\sin x + 2 = (\sin x + 1)(\sin x + 2)$ $\Rightarrow \boxed{R_f : [0, 6]}$

Min $\sin : 1 = (2)(3) = 6$

5 7

$\sin \; -\frac{b}{2a} = -\frac{3}{4} = \frac{9}{8} - \frac{9}{4} + 2 = \frac{7}{8}$

Max $\sin x : -1 = 2 - 3 + 2 = 1$

ب) $y = 2\sin^2 x + 3\sin x + 2$ $\Rightarrow \boxed{R_f : [\frac{7}{8}, 7]}$

Min $\sin : 1 = 2 + 3 + 2 = 7$

الف) $y = \sin^2 x + 4\underbrace{\cos^2 x}_{1-\sin^2 x} = \sin^2 x + 4 - 4\sin^2 x = \underbrace{-3\overbrace{\sin^2 x}^{[0,1]}}_{[-3,0]} + 4 = \begin{matrix} 4+0=4 \\ 4-3=1 \end{matrix} \Rightarrow \boxed{R_f : [1, 4]}$

5 8

ب) $y = \frac{2\cot x}{1+\cot^2 x} = \frac{\frac{2\cos x}{\sin x}}{\frac{1}{\sin^2 x}} = 2\sin x\cos x = \underbrace{\sin 2x}_{[-1,1]} \rightarrow \boxed{R_f : [-1, 1]}$

الف) $y = \sin^4 + \cos^4 = 1 - 2\sin^2 x\overbrace{\cos^2 x}^{1-\sin^2 x} = 1 - 2\sin^2 x + 2\sin^4 x = 2(\sin^2 x)^2 - 2\sin^2 + 1$

$\sin^2$	min 0	max 1	$-\frac{b}{2a} = \frac{2}{4} = \frac{1}{2}$
ضابطه	1	1	$\frac{2}{4} - \frac{2}{2} + 1 = \frac{1}{4}$

$\boxed{R_f : [\frac{1}{4}, 1]}$

1,75 9

ب) $y = [\sin^8 x + \cos^8 x] \rightarrow$

$2^{1-k} \le \sin^{2k} x + \cos^{2k} x \le 1$

$2^{-3} \le \sin^8 x + \cos^8 x \le 1 \quad \rightarrow k=4$

$[[\frac{1}{8}, 1]] \Rightarrow \boxed{R_f : [0, \cancel{1}]}$ $\{0, 1\}$

(۱)

الف) $y = \dfrac{۲x^۲ + ۷x - ۴}{x + ۴} = \rightsquigarrow x^۲ + ۷x - ۸ = (x + \cancel{۸})(x - ۱) = \dfrac{(x + \cancel{۴})(۲x - ۱)}{(x + \cancel{۴})} = ۲x - ۱$

$x \neq -۴$

$y \neq ۲(-۴) - ۱ \neq -۹$

۲

$\begin{cases} \mathbb{R}_f : \mathbb{R} \\ y \neq -۹ \end{cases}$

$\boxed{\mathbb{R}_f : \mathbb{R} - \{-۹\}}$

$x \neq$

د) $y = \dfrac{x^3-1}{x-1} = \dfrac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}} = x^2+x+1$

$-\dfrac{\Delta}{4a} = \dfrac{-(1-4)}{4} = \dfrac{3}{4} \rightarrow$

$\boxed{R_f : [\frac{3}{4}, +\infty)}$

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