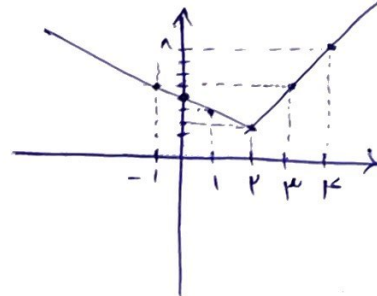


$$\text{الف) } y = x + |2x - 3|$$

$$2x = 3 \Rightarrow x = \frac{3}{2}$$

$$\begin{array}{c|c} & \frac{3}{2} \\ \hline x - 2x + 3 & x + 2x - 3 \\ \hline -x + 3 & 3x - 3 \end{array}$$

$$\begin{array}{c|c|c|c|c} x & \frac{3}{2} & 1 & 0 & -1 \\ \hline y & \frac{9}{2} & 5 & 3 & 1 \end{array} \quad \begin{array}{c|c|c|c|c} x & \frac{3}{2} & 1 & 0 & -1 \\ \hline y & \frac{9}{2} & 5 & 3 & 1 \end{array}$$



$$R = [\frac{3}{2}, +\infty)$$

$$\text{ب) } y = |x - 2| + |x - 1| - |x|$$

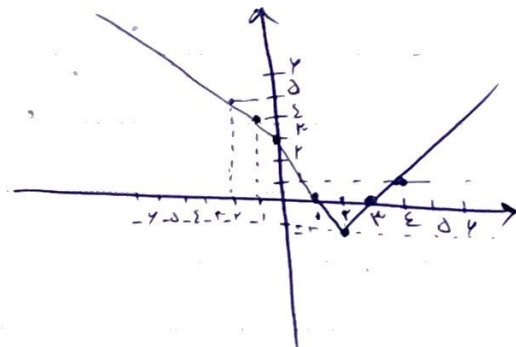
$$x = 2$$

$$x = 1$$

$$x = 0$$

$$\begin{array}{c|c|c|c|c} & 0 & 1 & 2 \\ \hline -x + 2 & -2x + 2 & -x + 1 & x - 2 \\ \hline & \oplus & \ominus & \ominus \end{array}$$

$$\begin{array}{c|c|c|c|c} x & -1 & -2 & & \\ \hline y & 3 & 2 & & \end{array} \quad \begin{array}{c|c|c|c|c} x & 2 & 1 & & \\ \hline y & 0 & 1 & & \end{array}$$



$$R = [-1, +\infty)$$

$$|2x - 3| - |x + 2| = k$$

$$x = 1$$

$$x = -2$$

$$\begin{array}{c|c|c|c|c} & -2 & & 1 & \\ \hline -2x + 3 + x + 2 & 2x + 3 - x - 2 & 2x - 3 - x - 2 \\ \hline -x + 3 & x + 1 & x - 5 \\ \hline & \oplus & \ominus & \ominus \end{array}$$

$$\min y = x = 1$$

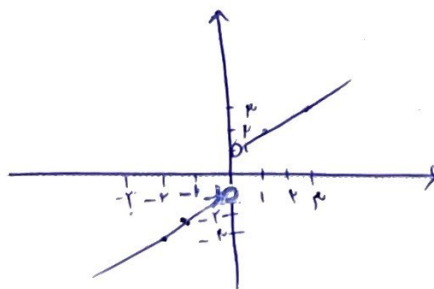
$$f(1) = |2 - 3| - |1 + 2| = |0| - |3| = -3 \quad k = -3$$

$$\text{الف) } y = x + \frac{x}{|x|}$$

$$\begin{array}{c|c} & 0 \\ \hline x - 1 & x + 1 \end{array}$$

$$\begin{array}{c|c|c|c|c} x & -1 & -2 & & \\ \hline y & -2 & -3 & & \end{array}$$

$$\begin{array}{c|c|c|c|c} x & 1 & 2 & & \\ \hline y & 2 & 3 & & \end{array}$$

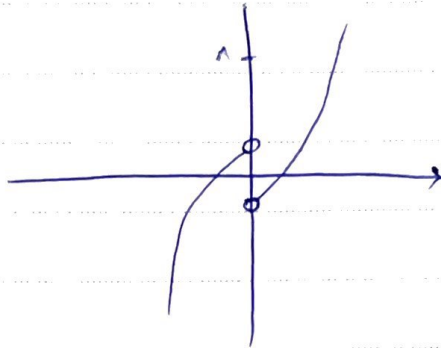


ب) $y = x|x| - \frac{x}{|x|}$

$\frac{-x^2+1}{-x^2+1} \mid \frac{x^2-1}{x^2-1}$

$\frac{x}{y} \mid \frac{-1-r-r}{0-r-1}$

$\frac{x}{y} \mid \frac{1-r-r}{0-r-1}$



-1

د) $y = [x] - 2x$

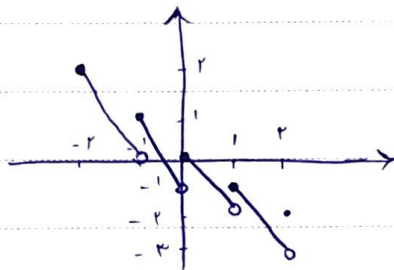
$x = 2 \rightarrow y = -2$

$1 \leq x < 2 \rightarrow 1 - 2x$

$0 \leq x < 1 \rightarrow -2x$

$-1 \leq x < 0 \rightarrow -2x - 1$

$-2 \leq x < -1 \rightarrow -2x - 1$



ب) $[x]|x|$

$x = 2 \rightarrow y = 2$

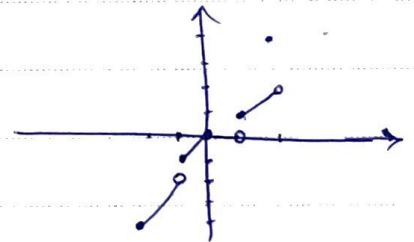
$x = -2 \rightarrow y = -2$

$1 \leq x < 2 \rightarrow |x|$

$0 \leq x < 1 \rightarrow 0$

$-1 \leq x < 0 \rightarrow -|x|$

$-2 \leq x < -1 \rightarrow -|x|$



الف) $y = 2x - [2x]$

$0 \leq x - [x] < 1$

$-1 \leq [x] - x \leq 0$

$Ry = [0, 1)$

ب) $y = \{x - \varepsilon[x]\}$

$\varepsilon(x - [x]) \rightarrow [0, \varepsilon)$

$Ry = [0, \varepsilon)$

-a

ج) $y = x - a\left[\frac{x}{a}\right]$

$a\left(\frac{x}{a} - \left[\frac{x}{a}\right]\right) = a\left(\frac{x}{a} - \left[\frac{x}{a}\right]\right)$

$Ry = [0, a)$

د) $y = [2x] - 2x$

$Ry = (-1, 0]$

الف) $y = 3\sin x + 4\cos x$

$$-\sqrt{9+16} \leq 3\sin x + 4\cos x \leq +\sqrt{9+16}$$

$$Ry = [-5, 5]$$

ب) $y = 2\sin x - 3\cos x$

$$-\sqrt{4+9} \leq 2\sin x - 3\cos x \leq \sqrt{4+9}$$

$$Ry = [-5, 5]$$

ج) $y = [2\sin x + 2\cos x]$

$$-\sqrt{4+4} \leq 2\sin x + 2\cos x \leq \sqrt{4+4}$$

$$[-\sqrt{2}, \sqrt{2}] \rightarrow Ry = \{y \mid y=2, -4 \leq y \leq 4\}$$

د) $y = \sqrt{\cos x - 3\sin x}$

$$-\sqrt{2} \leq \cos x - 3\sin x \leq \sqrt{2}$$

$$[-\sqrt{2}, \sqrt{2}] \rightarrow Ry = [0, \sqrt{2}]$$

هـ) $y = \sin^2 x + 3\sin x + 2$

$$-\frac{b}{2a} = -\frac{3}{2}, y = -\frac{1}{4}$$

$$x = -1 \rightarrow y = 0$$

$$x = 1 \rightarrow y = 4$$

$$Ry = [0, 4]$$

و) $y = 4\sin^2 x + 3\sin x + 2$

$$-\frac{b}{2a} = -\frac{3}{8}, y = \frac{1}{4}$$

$$x = -2 \rightarrow y = 1$$

$$x = 1 \rightarrow y = 4$$

$$Ry = [\frac{1}{4}, 4]$$

ز) $y = \sin^2 x + 2\cos^2 x$

$$\underbrace{\sin^2 + \cos^2}_1 + \underbrace{\cos^2}_{[0,1]}$$

$$0 \leq \cos^2 \leq 1 \rightarrow Ry = [1, 2]$$

ح) $y = \frac{\cos x}{1 + \cot^2 x}$

$$\frac{\cos x}{\sin^2 x} = \frac{\cos x \sin^2 x}{\sin^2 x} = \cos x \sin x$$

$$\sin^2 x \rightarrow$$

$$Ry = [-1, 1]$$

ط) $y = \sin^4 x + \cos^4 x$

$$1 - \frac{1}{2} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow \frac{1}{2} \leq \sin^4 x + \cos^4 x \leq 1$$

$$Ry = [\frac{1}{2}, 1]$$

ث) $y = [\sin^2 x + \cos^2 x]$

$$1 - \frac{1}{2} \leq \sin^2 x + \cos^2 x \leq 1$$

$$\frac{1}{2} \leq \sin^2 x + \cos^2 x \leq 1$$

$$Ry = \{0, 1\}$$

Subject: _____
Date _____

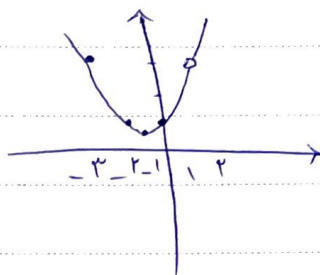
$$a) y = \frac{2x^2 + 7x - 2}{x + 2}$$

$$\frac{(x+2)(2x-1)}{(x+2)} = 2x-1$$

$$x \neq -2 \rightarrow y \neq -4$$

$$Ry = R - \{-4\}$$

$$b) y = \frac{x^2 - 1}{x - 1} = \frac{(x-1)(x+1)}{(x-1)} = x+1$$



$$x \neq 1, y \neq 2$$

$$Ry = \left[\frac{1}{2}, +\infty \right)$$