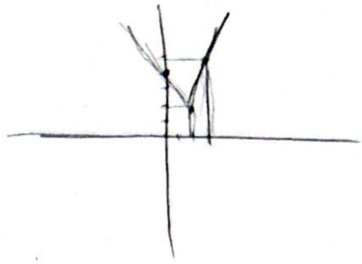
الیسا ایلالی – تکلیف شماره ۱

۱-

الف) $y = x + |2x - 2|$ $R_f = [2, +\infty)$

۱	
$x - 2x + 2$	$x + 2x - 2$
$-x + 2$	$3x - 2$

$\begin{vmatrix} 2 \\ 2 \end{vmatrix}$ $\begin{vmatrix} 3 \\ 7 \end{vmatrix}$ $\begin{vmatrix} 0 \\ 2 \end{vmatrix}$

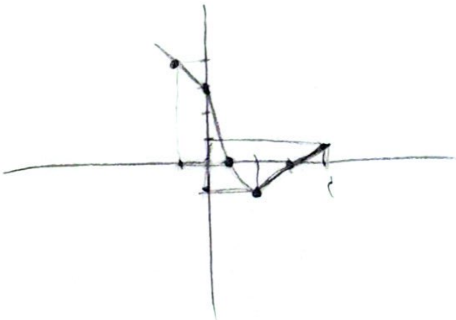


ب) $y = |x-2| + |x-1| - |x|$

۰	۱	۲	
$2 - x + 1 - x + x$	$2 - x + 1 - x - x$	$2 - x + x - 1 - x$	$x - 2 + x - 1 - x$
$-x + 3$	$-3x + 3$	$-x + 1$	$x - 3$

$\begin{vmatrix} 0 \\ 3 \end{vmatrix}$ $\begin{vmatrix} 1 \\ 0 \end{vmatrix}$ $\begin{vmatrix} 2 \\ -1 \end{vmatrix}$ $\begin{vmatrix} 3 \\ 0 \end{vmatrix}$ $\begin{vmatrix} -1 \\ 4 \end{vmatrix}$

$R_f = [-1, +\infty)$

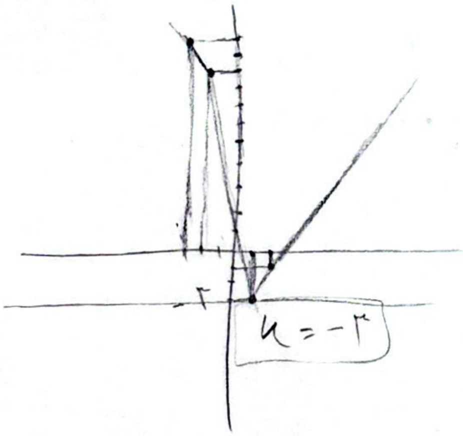


۲-

$|3x - 3| - |x + 2| = k$

-۲	۱	
$-3x + 3 + x + 2$	$3 - 3x - x - 2$	$3x - 3 - x - 2$
$-2x + 5$	$-4x + 1$	$2x - 5$

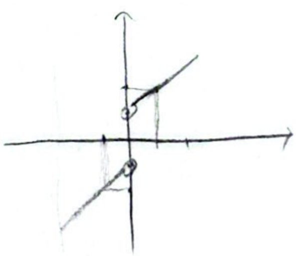
$\begin{vmatrix} 1 \\ -3 \end{vmatrix}$ $\begin{vmatrix} -2 \\ 9 \end{vmatrix}$ $\begin{vmatrix} 2 \\ -1 \end{vmatrix}$ $\begin{vmatrix} -3 \\ 11 \end{vmatrix}$



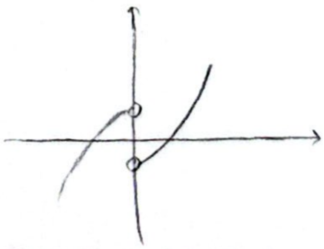
$k = -3$

۳-

الف) $x + \frac{x}{|x|}$ $\begin{cases} x + 1 & x > 0 \\ x - 1 & x < 0 \end{cases}$



ب) $y = x|x| - \frac{x}{|x|}$ $\begin{cases} x^2 - 1 & x > 0 \\ -x^2 + 1 & x < 0 \end{cases}$



۴-

الف) $y = [x] - 2x$

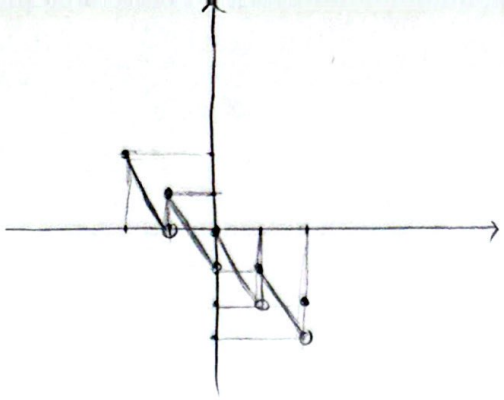
$-2 \le x < -1 \rightarrow -2 - 2x$

$-1 \le x < 0 \quad -1 - 2x$

$0 \le x < 1 \quad -2x$

$1 \le x < 2 \quad 1 - 2x$

$x = 2 \quad 2 - 2x$



ب) $y = [x]|x|$

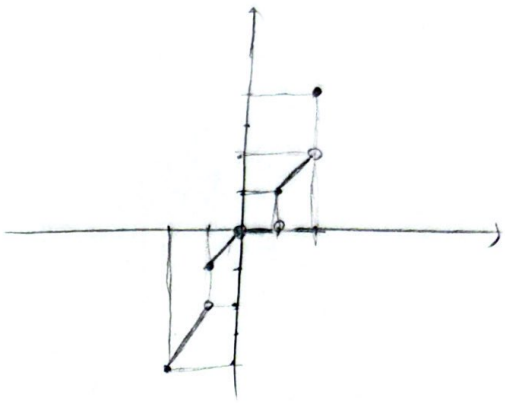
$-2 \le x < -1 \rightarrow 2x$

$-1 \le x < 0 \rightarrow x$

$0 \le x < 1 \rightarrow 0$

$1 \le x < 2 \rightarrow x$

$x = 2 \rightarrow 2x$



۵-

الف) $y = 2x - [2x]$ $\quad R_f = [0, 1)$

ب) $y = 2x - 2[x]$ $\quad 2\underbrace{(x - [x])}_{[0,1)} \Rightarrow R_f = [0, 2)$

ج) $y = x - 5\left[\frac{x}{5}\right]$ $\quad 5\underbrace{\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right)}_{[0,1)} \Rightarrow R_f = [0, 5)$

د) $y = [2x] - 2x$ $\quad R_f = (-1, 0]$

۶-

الف) $3\sin x + 2\cos x$ $\quad -\sqrt{9+4} \le 3\sin x + 2\cos x \le \sqrt{9+4}$ $\quad R_f = [-\sqrt{13}, \sqrt{13}]$

ب) $4\sin x - 3\cos x$ $\quad -\sqrt{16+9} \le 4\sin x - 3\cos x \le \sqrt{16+9}$ $\quad R_f = [-5, 5]$

ج) $[3\sin x + 5\cos x]$ $\quad -\sqrt{9+25} \le 3\sin x + 5\cos x \le \sqrt{25+25}$

$-\sqrt{34} \le 3\sin x + 5\cos x \le \sqrt{50}$

$\underbrace{-6}_{-5} \le 3\sin x + 5\cos x \le 7$ $\quad R_f = \{-6, -5, \ldots, 7\}$

د) $\sqrt{\cos x - 2\sin x}$ $\quad -\sqrt{1+4} \le \cos x - 2\sin x \le \sqrt{1+4}$

$-\sqrt{5} \le \cos x - 2\sin x \le \sqrt{5} \rightarrow [-\sqrt{5}, \sqrt{5}] \rightarrow [0, \sqrt[4]{5}]$

-۷

الف) $y = \sin^2 x + 2\sin x + 2$

$1 \to 1 + 2 + 2 = 5$ max

$-1 \to 1 - 2 + 2 = 1$

$-\frac{b}{2a} : \frac{-2}{2}$ ✗

$R_f = [0, 5]$

ب) $y = 2\sin^2 x + 2\sin x + 2$

$1 \to 2 + 2 + 2 = 6$ max

$-1 \to 2 - 2 + 2 = 2$

$-\frac{2}{4} \to \frac{+9}{8} - \frac{9}{4} + \frac{16}{8} = \frac{-9}{8} + \frac{16}{8} = \frac{7}{8}$ min

$R_f = [\frac{7}{8}, 6]$

-۸

الف) $y = \sin^2 x + 2\cos^2 x = \sin^2 x + \cos^2 x + \cos^2 x = 1 + \cos^2 x$

$1 \to 1 + 0 = 1$

$-1 \to 1 + 1 = 2$

$0 \to 1$

$R_f = [1, 2]$

ب) $y = \frac{2\cot x}{1 + \cot^2 x} = \frac{2\cot x}{\frac{1}{\sin^2 x}} = \frac{2\cos x}{\frac{1}{\sin x}} = 2\sin x \cos x = \sin 2x$ $\quad R_f = [-1, 1]$

-۹

الف) $y = \sin^2 x + \cos^2 x$

$2^{-2} \le \sin^2 x + \cos^2 x \le 1$

$2^{-2} = \frac{1}{4}$

$R_f = [\frac{1}{4}, 1]$

ب) $y = [\sin^8 x + \cos^8 x]$

$2^{-3} \le \sin^8 x + \cos^8 x < 1$

$\frac{1}{8} \quad 1$

$R_f = \{0, 1\}$

-۱۰

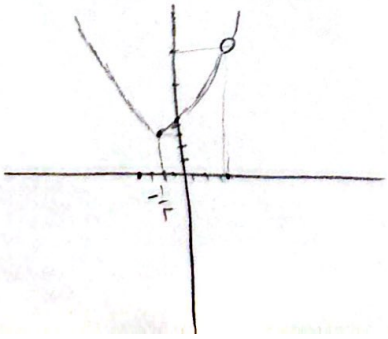
الف) $y = \frac{2x^2 + 7x - 4}{x + 4} = \frac{(2x - 1)(x + 4)}{x + 4} = 2x - 1$

$R_f = \mathbb{R} - \{-9\}$

$2x^2 + 7x - 4 = (2x - 1)(x + \frac{4}{2}) = (2x - 1)(x + 2)$

ب) $y = \frac{x^3 - 1}{x - 1} = \frac{(x - 1)(x^2 + x + 1)}{x - 1} = x^2 + x + 1$

min $\left| \begin{matrix} -\frac{1}{2} \\ \frac{3}{4} \end{matrix} \right.$



$R_f = [\frac{3}{4}, +\infty)$