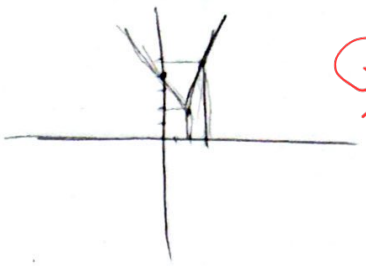
۱- الف) $y = x + |2x - 2|$

$R_f = [2, +\infty)$

x	$-\infty$		1		$+\infty$
$2x-2$	$-$	0	$+$		$+$
y	$-x+2$		$3x-2$		

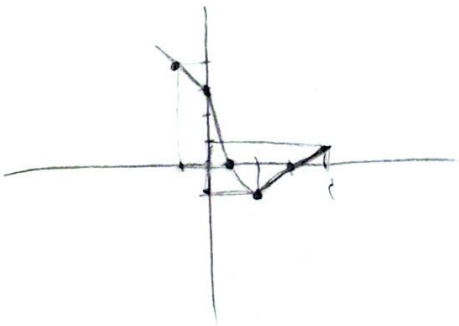
x	2	1	0
y	0	1	2



ب) $y = |x-2| + |x-1| - |x|$

x	$-\infty$		0		1		2		$+\infty$
$x-2$	$-$	$-$	$-$	$-$	$-$	0	$+$		$+$
$x-1$	$-$	$-$	0	$+$		$+$		$+$	
$-x$		$+$		$+$		$+$		$+$	
y	$-x+3$		$-x+2$		$-x+1$		$x-2$		

x	0	1	2	3
y	3	2	1	0

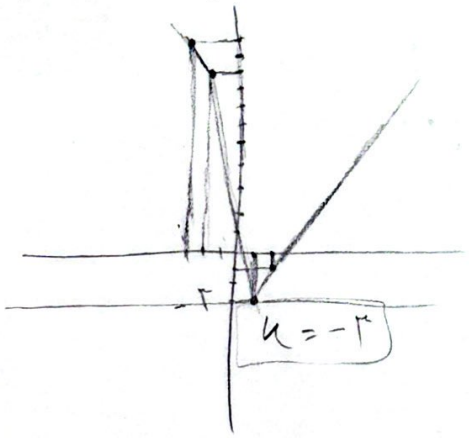


$R_f = [-1, +\infty)$

۲- $|3x-3| - |x+2| = k$

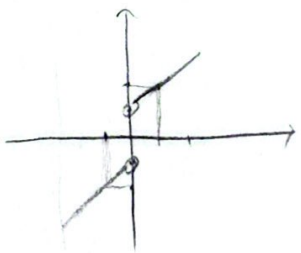
x	$-\infty$		-2		1		$+\infty$
$3x-3$	$-$	$-$	$-$	0	$+$		$+$
$-x-2$	$+$		$+$		$+$		$+$
y	$-2x+1$		$-x-5$		$2x-5$		

x	-2	1	2	3
y	9	-3	-1	1



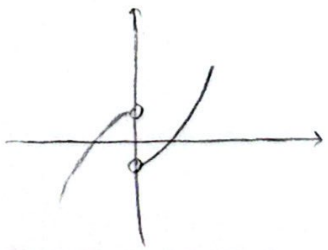
۱- الف) $x + \frac{x}{|x|}$

$$\begin{cases} x+1 & x > 0 \\ x-1 & x < 0 \end{cases}$$



ب) $y = x|x| - \frac{x}{|x|}$

$$\begin{cases} x^2 - 1 & x > 0 \\ -x^2 + 1 & x < 0 \end{cases}$$



۴-

الف) $y = [x] - 2x$

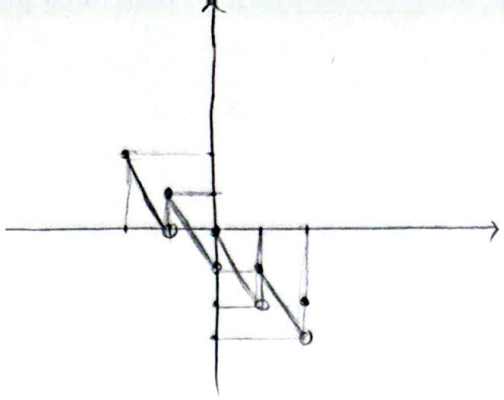
$-2 \le x < -1 \rightarrow -2 - 2x$

$-1 \le x < 0 \quad -1 - 2x$

$0 \le x < 1 \quad -2x$

$1 \le x < 2 \quad 1 - 2x$

$x = 2 \quad 2 - 2x$



ب) $y = [x]|x|$

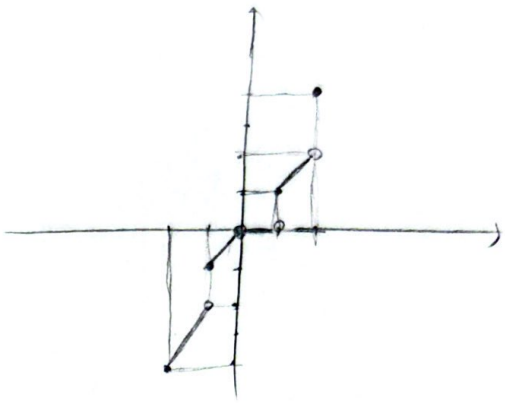
$-2 \le x < -1 \rightarrow 2x$

$-1 \le x < 0 \rightarrow x$

$0 \le x < 1 \rightarrow 0$

$1 \le x < 2 \rightarrow x$

$x = 2 \rightarrow 2x$



۵-

الف) $y = 2x - [2x]$ $\quad R_f = [0, 1)$

ب) $y = 2x - 2[x]$ $\quad 2\left(\underbrace{x - [x]}_{[0,1)}\right) \Rightarrow R_f = [0, 2)$

ج) $y = x - 3\left[\frac{x}{3}\right]$ $\quad 3\left(\underbrace{\frac{x}{3} - \left[\frac{x}{3}\right]}_{[0,1)}\right) \Rightarrow R_f = [0, 3)$

د) $[2x] - 2x$ $\quad R_f = (-1, 0]$

۶-

الف) $3\sin x + 2\cos x$ $\quad -\sqrt{9+4} \le 3\sin x + 2\cos x \le \sqrt{9+4}$ $\quad R_f = [-\sqrt{13}, \sqrt{13}]$

ب) $4\sin x - 3\cos x$ $\quad -\sqrt{16+9} \le 4\sin x - 3\cos x \le \sqrt{16+9}$ $\quad R_f = [-5, 5]$

ج) $[3\sin x + 5\cos x]$ $\quad -\sqrt{9+25} \le 3\sin x + 5\cos x \le \sqrt{25+25}$

$-\underbrace{\sqrt{49}}_{-7} \le 3\sin x + 5\cos x \le \sqrt{49}$ $\quad R_f = \{-7, -6, \ldots, 6, 7\}$

د) $\sqrt{\cos x - 2\sin x}$ $\quad -\sqrt{1+4} \le \cos x - 2\sin x \le \sqrt{1+4}$

$-\sqrt{5} \le \cos x - 2\sin x \le \sqrt{5} \rightarrow [-\sqrt{5}, \sqrt{5}] \cap [0, \sqrt{5}]$

-۷

الف) $y = \sin^2 x + 2\sin x + 2$; $1 + 2 + 2 = 5$ max ; $-1 \to 1 - 2 + 2 = 1$; $\frac{-b}{2a} = \frac{-2}{2}$ ✗

$R_f = [1, 5]$

ب) $y = 2\sin^2 x + 2\sin x + 2$; $1 \to 2 + 2 + 2 = 6$ max ; $-1 \to 2 - 2 + 2 = 2$; $-\frac{2}{4}$; $\frac{+9}{8} - \frac{9}{4} + \frac{14}{8} = \frac{-9}{8} + \frac{14}{8} = \frac{5}{8}$ min

$R_f = [\frac{5}{8}, 6]$

-۸

الف) $y = \sin^2 x + 2\cos^2 x = \sin^2 x + \cos^2 x + \cos^2 x = 1 + \cos^2 x$; $1 \to 1 + 1 = 2$; $-1 \to 1 + 1 = 2$; $0 \to 1$

$R_f = [1, 2]$

ب) $y = \frac{2\cot x}{1 + \cot^2 x} = \frac{2\cot x}{\frac{1}{\sin^2 x}} = \frac{2\cos x}{\frac{1}{\sin x}} = 2\sin x \cos x = \sin 2x$ $R_f = [-1, 1]$

-۹

الف) $y = \sin^2 x + \cos^2 x$; $1 - 2 \le \sin^2 x + \cos^2 x \le 1$; $2^{-2} = \frac{1}{4}$ $R_f = [\frac{1}{4}, 1]$

ب) $y = [\sin^4 x + \cos^4 x]$; $1 - 2 \le \sin^4 x + \cos^4 x < 1$; $\frac{1}{8} \quad 1$ $R_f = \{0, 1\}$

-۱۰

الف) $y = \frac{2x^2 + 7x - 4}{x + 4} = \frac{(2x - 1)(x + 4)}{x + 4} = 2x - 1$ $R_f = \mathbb{R} - \{-9\}$

$2x^2 + 7x - 4 = (2x - 1)(x + \frac{4}{2}) = (2x - 1)(x + 2)$

ب) $y = \frac{x^3 - 1}{x - 1} = \frac{(x - 1)(x^2 + x + 1)}{x - 1} = x^2 + x + 1$ min $\left| \begin{matrix} -\frac{1}{2} \\ \frac{3}{4} \end{matrix} \right.$

$R_f = [\frac{3}{4}, +\infty)$