

الف) $y = x^3 - 3x^2 + 3x \Rightarrow x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1$ (۱)

$y = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1$

$\rightarrow x = \sqrt[3]{y-1} + 1$

$R_y = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow y(x^2 - 2x) = 1 \rightarrow x^2 - 2x = \frac{1}{y}$ (۱)

$\rightarrow x^2 - 2x + 1 = \frac{1}{y} + 1 \rightarrow (x-1)^2 = \frac{1+y}{y} \rightarrow x-1 = \pm \sqrt{\frac{1+y}{y}}$

$\rightarrow x = \pm \sqrt{\frac{1+y}{y}} + 1$

$\begin{matrix} 1+y=0 \rightarrow y=-1 \\ y=0 \end{matrix}$

$\frac{-1}{+\infty - \frac{1}{y}} +$

$R_y = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 2x + 1$

$a > 0 \rightarrow \min \left| x = \frac{-b}{2a} = 1 \right.$

$y = 1 - 1 + 1 = 1$

$R_y = [1, +\infty)$

ب) $y = -x^2 + 4x + 3$

$a < 0 \rightarrow \max$

$\left| x = \frac{-b}{2a} = 2 \right.$

$-4 + 11 + 3 = 11$

$R_y = (-\infty, 11]$

ج) $y = \sqrt{x^2 - 2x - 3}$

$a > 0 \rightarrow \min$

$\left| x = \frac{-b}{2a} = 1 \right.$

$1 - 1 - 3 = -3$

عبارت زیر را در کمال به همواره نامنفی است

$\sqrt{[-3, +\infty)} = [0, +\infty)$

$\rightarrow R_y = [0, +\infty)$

د) $y = \sqrt{4x - x^2} = \sqrt{-x^2 + 4x}$

$a < 0 \rightarrow \max$

$\left| x = \frac{-b}{2a} = 2 \right.$

$11 - 9 = 9$

عبارت زیر را در کمال همواره نامنفی است

$\sqrt{(-\infty, 9]} = [0, 3]$

$\rightarrow R_y = [0, 3]$

الف) $y = x^3 - 2x^2 + 3x + 1$

$R_y = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 3x + 2$

$R_y = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 3x + 1}$

$\sqrt{R} = [0, +\infty) \rightarrow R_y = [0, +\infty)$

عبارت زیر را در یکال همواره نامنفی

د) $y = (x^3 - 4x^2 + 3x + 1)^2$

$\sqrt{R} = [0, +\infty) \rightarrow R_y = [0, +\infty)$

عبارت توان دو همواره نامنفی

الف) $y = \frac{3x+1}{x-2}$

$R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_y = \mathbb{R} - \{3\}$

ب) $y = \frac{2x+1}{x+1}$

$R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_y = \mathbb{R} - \{1\}$

الف) $y = \sqrt{\frac{2x+1}{x+1}}$

$R_y = \sqrt{\mathbb{R} - \left\{\frac{a}{c}\right\}} \rightarrow R_y = \sqrt{\mathbb{R} - \{1\}}$

عبارت همواره نامنفی

$\rightarrow R_y = [0, +\infty) - \{\sqrt{1}\}$

ب) $y = \sqrt{\frac{3x+1}{1-x}} = \sqrt{\frac{3x+1}{-x+1}}$

$R_y = \sqrt{\mathbb{R} - \left\{\frac{a}{c}\right\}} \rightarrow R_y = \sqrt{\mathbb{R} - \{-1\}}$

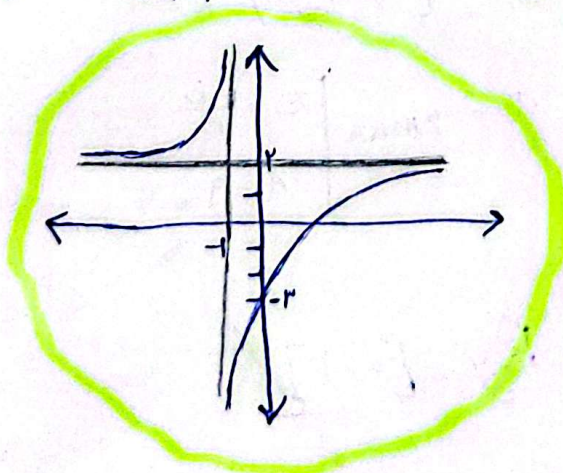
$\rightarrow R_y = [0, +\infty)$

الف) $y = \frac{2x-3}{x+1}$

$x=0 \rightarrow y = -3$

$\frac{a}{c} = \frac{2}{1} = 2$

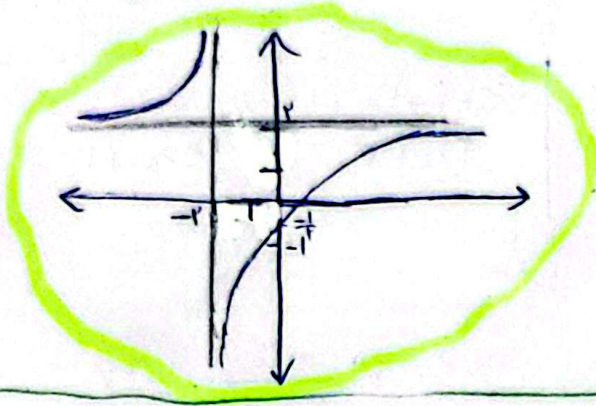
$x+1=0 \rightarrow x=-1$



$$1) y = \frac{x-1}{x+1} \xrightarrow{x=0} y = \frac{-1}{1}$$

$$\frac{a}{c} = \frac{1}{1} = 1$$

$$x+1=0 \rightarrow x=-1$$



$$2) y = \sin x + \frac{1}{\sin x}$$

$$-1 \leq \sin x \leq 1 \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$$

$$3) y = \frac{x^2+1}{x^2} = x^2 + \frac{1}{x^2}$$

$$R_y = (-\infty, -1] \cup [1, +\infty)$$

$$4) y = \frac{\sqrt{x^2}+1}{\sqrt{x^2}} = \sqrt{x^2} + \frac{1}{\sqrt{x^2}}$$

$$R_y = (-\infty, 1] \cup [1, +\infty)$$

$$5) y = \sqrt{x^2} + \frac{1}{\sqrt{x^2}}$$

$$R_y = [1, +\infty)$$

$$6) y = x^2 + \frac{1}{x^2+c} \rightarrow y = x^2 + \frac{1}{x^2+c} + c - c$$

$$\rightarrow y = \underbrace{x^2+c}_{\frac{x^2+c}{1}} + \frac{1}{\frac{x^2+c}{1}} - c$$

$$x^2+c \xrightarrow{x=0} 0+c=c$$

$$\rightarrow y = c + \frac{1}{c} - c = \frac{1}{c}$$

$$R_y = [\frac{1}{c}, +\infty)$$

$$7) y = \frac{x^2+a}{\sqrt{x^2+\varepsilon}} = \underbrace{\sqrt{x^2+\varepsilon}}_x + \frac{1}{\sqrt{x^2+\varepsilon}}$$

$$\rightarrow y = x + \frac{1}{x} = \frac{a}{r}$$

$$\sqrt{x^2+\varepsilon} \xrightarrow{x=0} \sqrt{\varepsilon} = r$$

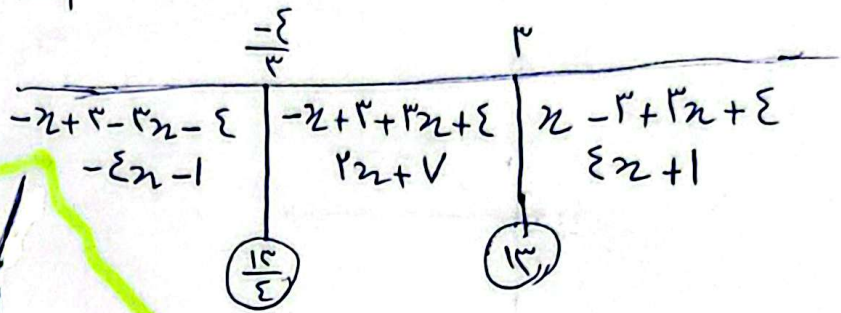
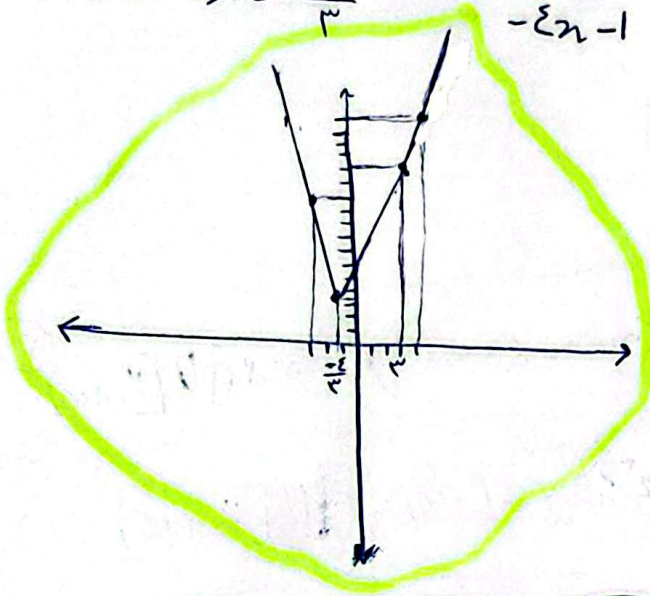
$$R_y = [\frac{a}{r}, +\infty)$$

(9)

$$y = |x-3| + |3x+2|$$

$$x-3=0 \rightarrow x=3$$

$$3x+2=0 \rightarrow x=-\frac{2}{3}$$



$$y = 2x+1 \xrightarrow{x=3} y=7$$

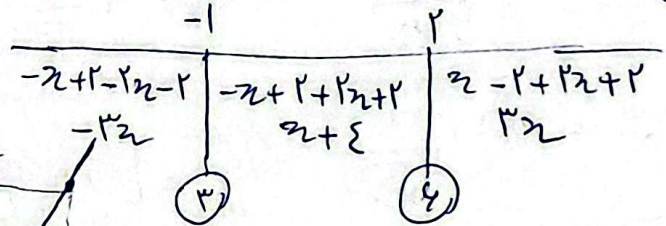
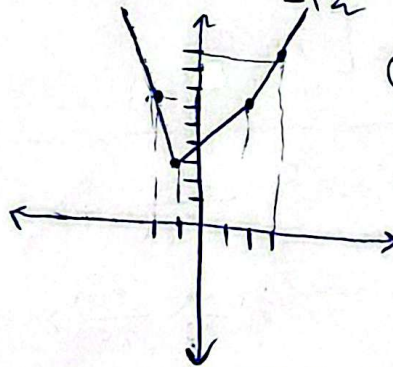
$$y = -x-1 \xrightarrow{x=-2/3} y=1$$

$$a) y = |x-2| + |2x+2|$$

$$x-2=0 \rightarrow x=2$$

$$2x+2=0 \rightarrow x=-1$$

$$Ry = [1, +\infty)$$



$$y = x+2 \xrightarrow{x=2} y=4$$

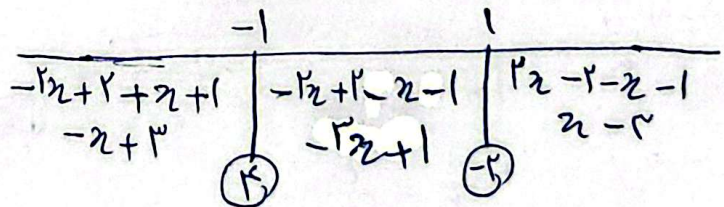
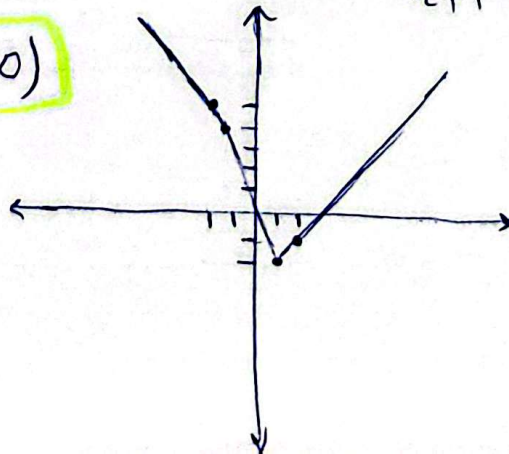
$$y = -3x \xrightarrow{x=-1} y=3$$

$$b) y = |2x-2| - |x+1|$$

$$2x-2=0 \rightarrow x=1$$

$$x+1=0 \rightarrow x=-1$$

$$Ry = [-1, +\infty)$$



$$y = x-1 \xrightarrow{x=1} y=0$$

$$y = -x+1 \xrightarrow{x=-1} y=2$$