

الف) $y = x^3 - 3x^2 + 3x \Rightarrow x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1$ (۱)

$y = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1$ (۲)

$\rightarrow x = \sqrt[3]{y-1} + 1$

$R_y = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow y(x^2 - 2x) = 1 \rightarrow x^2 - 2x = \frac{1}{y}$ (۱)

$\rightarrow x^2 - 2x + 1 = \frac{1}{y} + 1 \rightarrow (x-1)^2 = \frac{1+y}{y} \rightarrow x-1 = \pm \sqrt{\frac{1+y}{y}}$

$\rightarrow x = \pm \sqrt{\frac{1+y}{y}} + 1$

$\frac{1+y}{y} = 0 \rightarrow y = -1$
 $y = 0$

$\frac{-1}{+0-0} +$

$R_y = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 2x + 1$

$a > 0 \rightarrow \min \left| x = \frac{-b}{2a} = 1 \right.$

$y = 1 - 2 + 1 = 0$

$R_y = [0, +\infty)$

ب) $y = -x^2 + 4x + 3$

$a < 0 \rightarrow \max$

$\left| x = \frac{-b}{2a} = 2 \right.$

$-4 + 18 + 3 = 17$

$R_y = (-\infty, 17]$

ج) $y = \sqrt{x^2 - 2x - 3}$

$a > 0 \rightarrow \min$

$\left| x = \frac{-b}{2a} = 1 \right.$

$1 - 2 - 3 = -4$

عبارت زیر را در کمال به همواره نامنفی است

$\sqrt{[-4, +\infty)} = [0, +\infty)$

$\rightarrow R_y = [0, +\infty)$

د) $y = \sqrt{4x - x^2} = \sqrt{-x^2 + 4x}$

$a < 0 \rightarrow \max$

$\left| x = \frac{-b}{2a} = 2 \right.$

$11 - 9 = 2$

عبارت زیر را در کمال به همواره نامنفی است

$\sqrt{(-\infty, 4]} = [0, 2]$

$\rightarrow R_y = [0, 2]$

$$الف) y = x^3 - 2x^2 + 2x + 1$$

$$R_y = \mathbb{R}$$

ج) ٥

$$ب) y = x^4 - 4x^3 + 3x + 2$$

$$R_y = \mathbb{R}$$

ج) ٥

$$ج) y = \sqrt{x^4 - 4x^3 + 5x + 1}$$

$$\sqrt{R} = [0, +\infty) \rightarrow R_y = [0, +\infty)$$

ج) ٥

$$د) y = (x^5 - 4x^3 + 3x + 1)^2$$

$$\sqrt{R} = [0, +\infty) \rightarrow R_y = [0, +\infty)$$

ج) ٥

$$الف) y = \frac{3x+1}{x-2}$$

$$R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_y = \mathbb{R} - \{3\}$$

ج) ٥

$$ب) y = \frac{2x+1}{x+1}$$

$$R_y = \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_y = \mathbb{R} - \{1\}$$

ج) ٥

$$الف) y = \sqrt{\frac{2x+5}{x+1}}$$

$$R_y = \sqrt{\mathbb{R} - \left\{\frac{a}{c}\right\}} \rightarrow R_y = \sqrt{\mathbb{R} - \{1\}}$$

ج) ٥

$$R_y = [0, +\infty) - \{\sqrt{1}\}$$

$$ب) y = \sqrt{\frac{3x+1}{1-x}} = \sqrt{\frac{3x+1}{-x+1}}$$

$$R_y = \sqrt{\mathbb{R} - \left\{\frac{a}{c}\right\}} \rightarrow R_y = \sqrt{\mathbb{R} - \{-1\}}$$

ج) ٥

$$\rightarrow R_y = [0, +\infty)$$

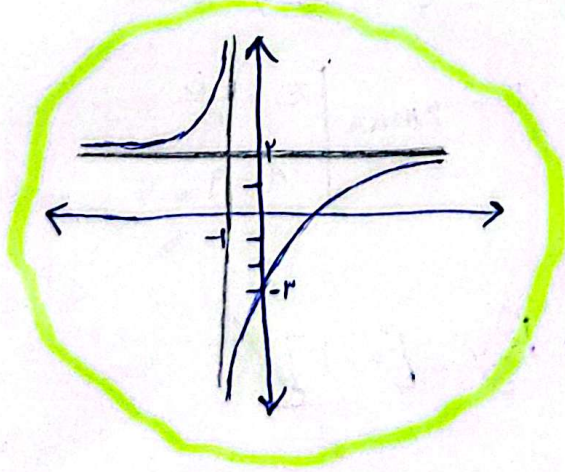
$$الف) y = \frac{2x-3}{x+1}$$

$$x=0 \rightarrow y = -3$$

$$\frac{a}{c} = \frac{2}{1} = 2$$

ج) ٥

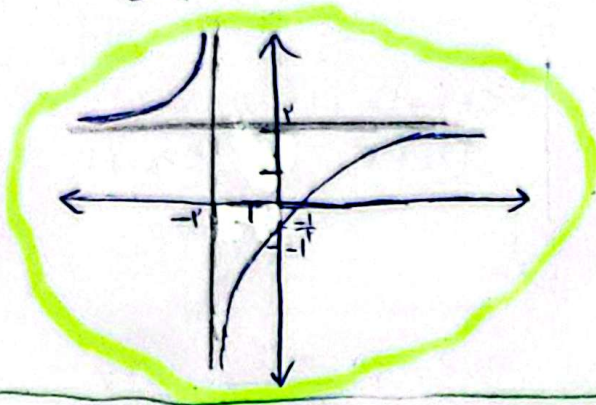
$$x+1=0 \rightarrow x = -1$$



$$1) y = \frac{x-1}{x+1} \xrightarrow{x=0} y = \frac{-1}{1}$$

$$\frac{a}{c} = \frac{1}{1} = 1$$

$$x+1=0 \rightarrow x=-1$$



$$2) y = \sin x + \frac{1}{\sin x}$$

$$-1 \leq \sin x \leq 1 \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$$

$$3) y = \frac{x^2+1}{x^2} = x^2 + \frac{1}{x^2}$$

$$R_y = (-\infty, -1] \cup [1, +\infty)$$

$$4) y = \frac{\sqrt{x^2}+1}{\sqrt{x^2}} = \sqrt{x^2} + \frac{1}{\sqrt{x^2}}$$

$$R_y = (-\infty, 1] \cup [1, +\infty)$$

$$5) y = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$R_y = [1, +\infty)$$

$$6) y = x^2 + \frac{1}{x^2+c} \rightarrow y = x^2 + \frac{1}{x^2+c} + c - c$$

$$\rightarrow y = \underbrace{x^2+c}_{\frac{x^2+c}{1}} + \frac{1}{\frac{x^2+c}{1}} - c$$

$$x^2+c \xrightarrow{x=0} 0+c = c$$

$$\rightarrow y = c + \frac{1}{c} - c = \frac{1}{c}$$

$$R_y = [\frac{1}{c}, +\infty)$$

$$7) y = \frac{x^2+a}{\sqrt{x^2+\varepsilon}} = \underbrace{\frac{x^2+\varepsilon}{\sqrt{x^2+\varepsilon}}}_r + \frac{1}{\sqrt{x^2+\varepsilon}}$$

$$\rightarrow y = r + \frac{1}{r} = \frac{a}{r}$$

$$\sqrt{x^2+\varepsilon} \xrightarrow{x=0} \sqrt{\varepsilon} = r$$

$$R_y = [\frac{a}{r}, +\infty)$$

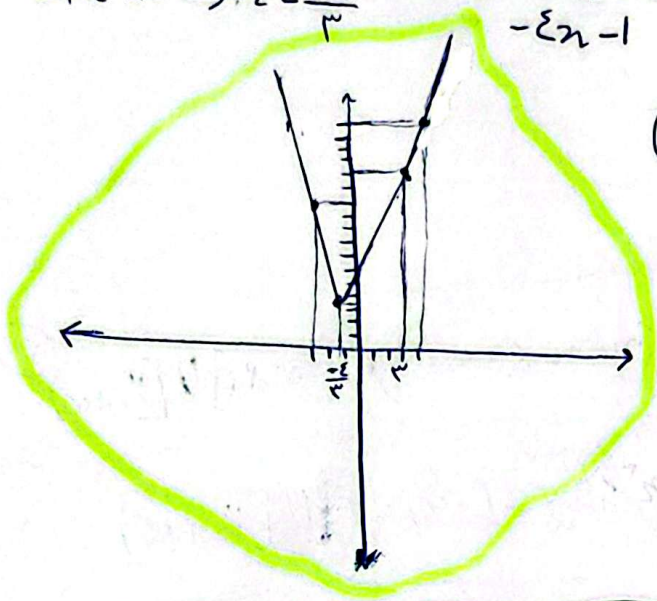
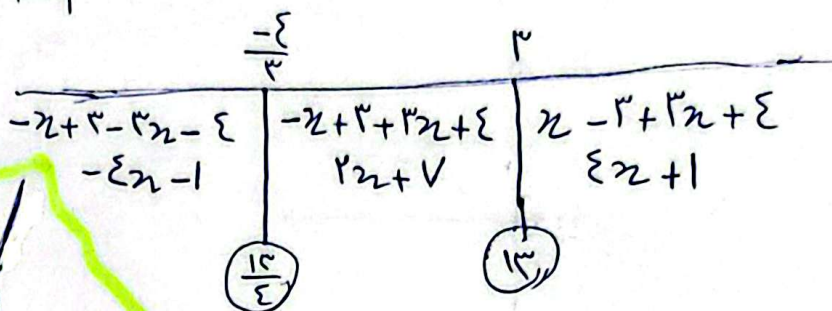
$$y = |x-3| + |3x+2|$$

9

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$$x-3=0 \rightarrow x=3$$

$$3x+2=0 \rightarrow x=-\frac{2}{3}$$



$$y = 4x-1 \xrightarrow{x=3} y = 11$$

$$y = -4x-4 \xrightarrow{x=-\frac{2}{3}} y = 10\frac{2}{3}$$

$$b) y = |x-1| + |2x+1|$$

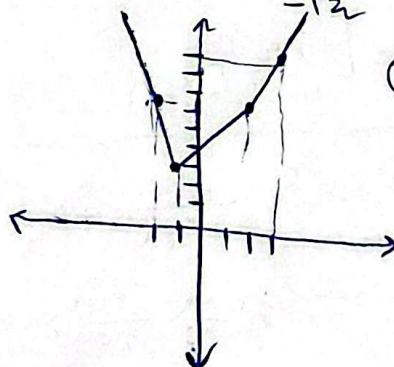
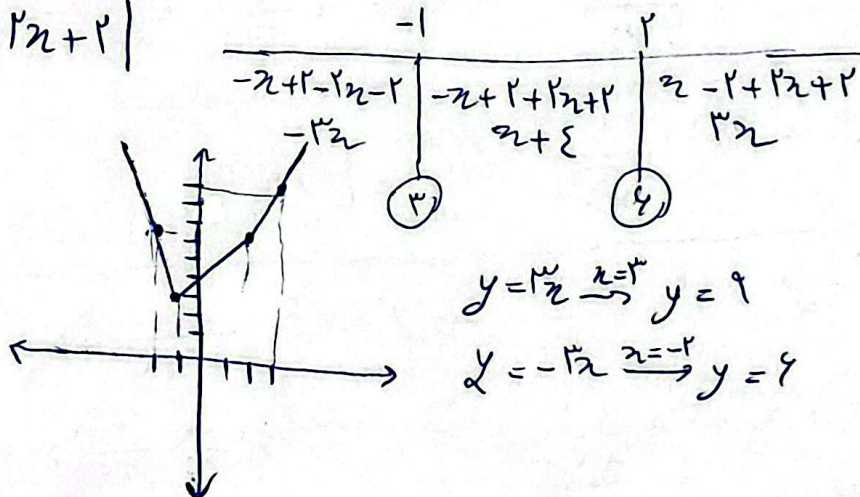
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$$x-1=0 \rightarrow x=1$$

$$2x+1=0 \rightarrow x=-\frac{1}{2}$$

$$Ry = [1, +\infty)$$



$$y = 3x \xrightarrow{x=1} y = 3$$

$$y = -3x-2 \xrightarrow{x=-\frac{1}{2}} y = 1.5$$

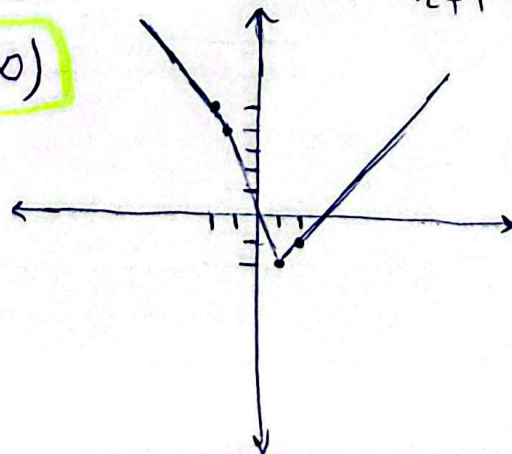
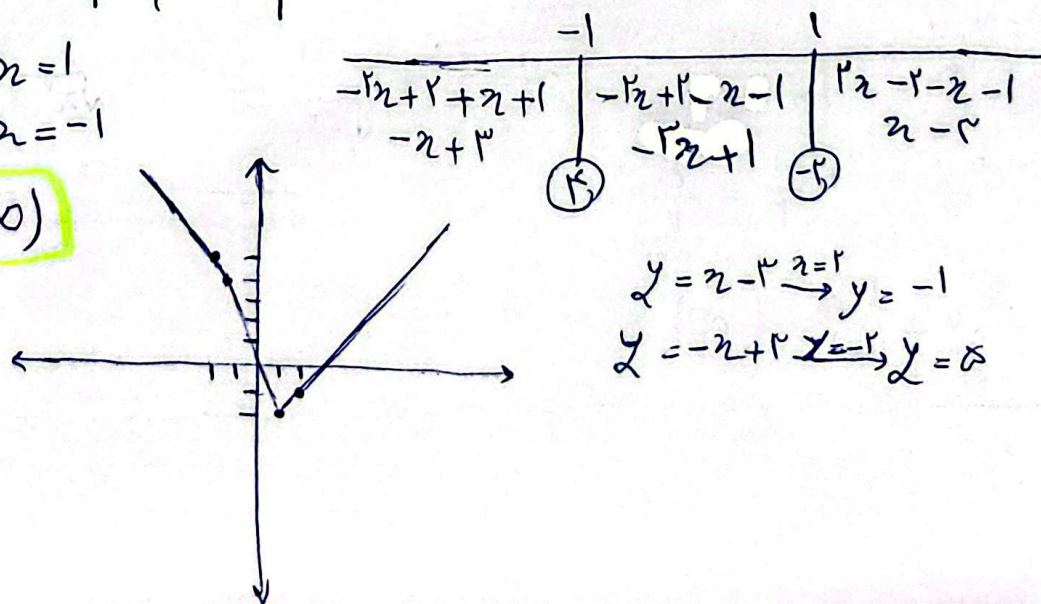
$$c) y = |2x-1| - |x+1|$$

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$$2x-1=0 \rightarrow x=\frac{1}{2}$$

$$x+1=0 \rightarrow x=-1$$

$$Ry = [-1, +\infty)$$



$$y = x-2 \xrightarrow{x=\frac{1}{2}} y = -1.5$$

$$y = -3x \xrightarrow{x=-1} y = 3$$