

1 الف) $y = x^3 - 3x^2 + 3x - 1 + 1$

2 $y - 1 = (x - 1)^3 \xrightarrow{\sqrt[3]{\quad}} \sqrt[3]{y - 1} = x - 1$

3 $\hookrightarrow x = \sqrt[3]{y - 1} + 1$ $R_f = \mathbb{R}$

5 ب) $y = \frac{1}{x^2 - 2x}$

$yx^2 - 2xy = 1 \rightarrow yx^2 - 2xy - 1 = 0$

$y(x^2 - 2x) = 1 \quad x^2 - 2x = \frac{1}{y}$

~~$x^2 - 2x + 1 = \frac{1}{y} + 1$~~

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8 $R_f = (-\infty, -1] \cup (0, +\infty)$ $(x-1)^2 \leq \frac{1}{y} + 1$

$x - 1 \leq \sqrt{\frac{1+y}{y}} \rightarrow x \leq \sqrt{\frac{1+y}{y}} + 1$

11 الف) $x^2 \leq x + 1 \Leftrightarrow y \leq -1 + 1 = 0$

12 $\left| \frac{-b}{2a} \rightarrow \frac{\pm \epsilon}{2} = y \right.$

$R_f = [4, +\infty)$

13 $\left| \frac{-\Delta}{4a} \rightarrow y \right.$

$a > 0 \rightarrow \text{parabole}$

15 ب) $y = -x^2 + 4x + 3$

16 $\left| \frac{-b}{2a} = \frac{-4}{-2} = 2 \right.$

$R_f = (-\infty, 12]$

17 $\left| \frac{-\Delta}{4a} = \frac{-4 + 16 + 3}{-2} = 12 \right.$

$a < 0 \rightarrow \text{max}$

$R_f = [0, +\infty)$

19 ج) $y = \sqrt{x^2 - 4x + 3}$ $\rightarrow x^2 - 4x + 3$

20 $\left| \frac{-b}{2a} = \frac{\pm \epsilon}{2} = y \right.$

$\sqrt{\quad} [-3, +\infty)$

$\left| \frac{-\Delta}{4a} = \frac{-4 + 16 + 3}{-2} = -3 \right.$ $a > 0$

$\rightarrow \text{min}$

$$>) y = \sqrt{4n - n^2} \rightarrow -n^2 + 4n$$

$$\left| \begin{array}{l} \frac{-b}{2a} \\ -\frac{\Delta}{4a} \end{array} \right. \Rightarrow \frac{-4}{-2} = 2 \quad \sqrt{(-\infty, 4]} \rightarrow R_f = [0, 2]$$

$$-4 + 16 \Rightarrow (4) \quad a < 0 \rightarrow \max_{x \in [a, b]}$$

$$\text{الف) } n^2 - 2n^2 + 2n + 1 = y \quad R_f \subseteq \mathbb{R}$$

$$\text{ب) } n^2 - 4n^2 + 2n + 1 = y \quad R_f \subseteq \mathbb{R}$$

$$\text{ج) } \sqrt{n^2 - 4n^2 + 2n + 1} \Rightarrow \sqrt{R} \Rightarrow [0, +\infty) = R_f$$

$$\text{د) } (n^2 - 4n^2 + 2n + 1)^2 \rightarrow (-\infty, +\infty)^2 \quad R_f \subseteq [0, +\infty)$$

$$\hookrightarrow (\mathbb{R})^2 \quad \hookrightarrow [0, +\infty)$$

$$\text{الف) } \frac{4n+1}{n-2} \quad \frac{a}{c} = 2$$

$$R_f \subseteq \mathbb{R} - \{2\}$$

$$\text{ب) } \frac{4n+1}{n+1} \quad \frac{a}{c} = \frac{4}{1} = 4$$

$$R_f \subseteq \mathbb{R} - \{4\}$$

Subject : _____

Date: _____

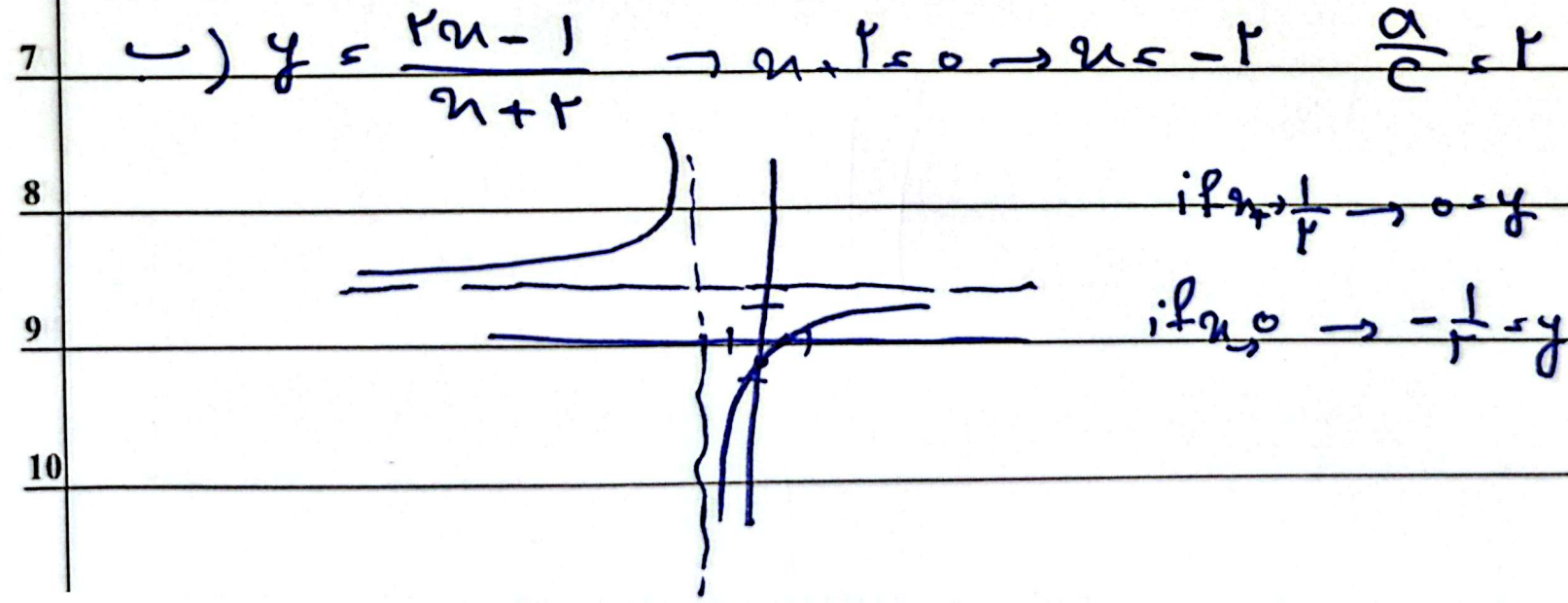
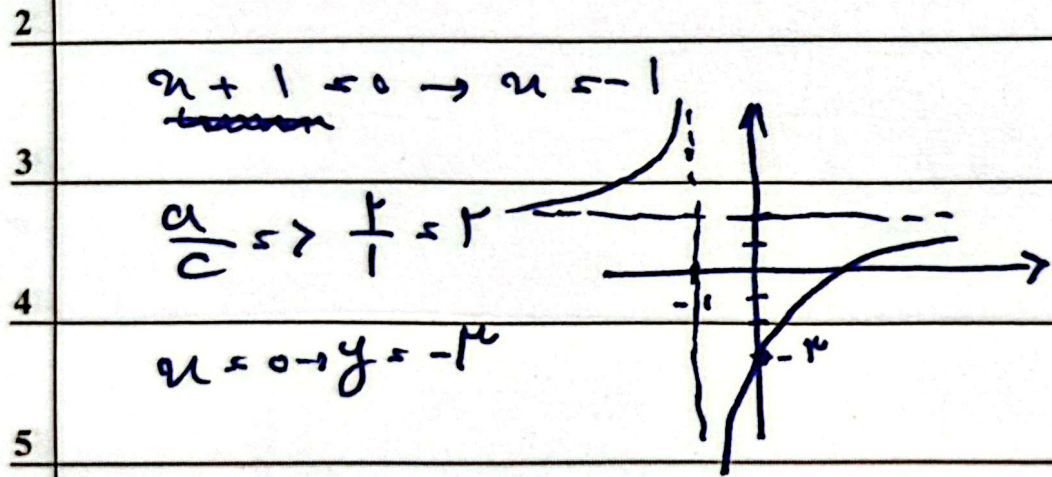
1 (أ) $y = \sqrt{\frac{2n+1}{n+1}} \rightarrow \sqrt{\mathbb{R} - \{1\}} \rightarrow [0, +\infty) - \{\sqrt{2}\} = R_f$ (د)

2

3 (ب) $y = \sqrt{\frac{n+1}{-n+1}} \rightarrow \sqrt{\mathbb{R} - \{-1\}} \rightarrow [0, +\infty) = R_f$

4

1 (ii) $\frac{rx - r}{x+1} \rightarrow \frac{ax+b}{cx+d}$



14 1) $y = \sin u + \frac{1}{\sin u} \rightarrow \sin u = a$ (✓)

15 $\hookrightarrow a + \frac{1}{a} \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$

16 2) $\frac{u^r + 1}{u^r} \rightarrow \frac{u^r}{u^r} + \frac{1}{u^r} \rightarrow u^r + \frac{1}{u^r} \rightarrow u^r = a'$

17 $\hookrightarrow a' + \frac{1}{a'} \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$

19 3) $\frac{\sqrt[r]{u^r} + 1}{\sqrt[r]{u^r}} \rightarrow \sqrt[r]{u} + \frac{1}{\sqrt[r]{u}} \rightarrow \sqrt[r]{u} = a''$

20 $\} a'' + \frac{1}{a''} \rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$

$$a) \sqrt{u} + \frac{1}{\sqrt{u}} \cdot y \rightarrow \boxed{R_y = [r, +\infty)}$$

$$b) y = \frac{u^r + 1}{u^r + \mu} \xrightarrow{+r-\mu} \left(\frac{u^r + \mu}{u^r + \mu} \right) - \mu$$

①

$$u \rightarrow 0 \rightarrow \mu + \frac{1}{\mu} - \mu \rightarrow \left(\frac{1}{\mu} \right) \quad \boxed{R_f = \left[\frac{1}{\mu}, +\infty \right)}$$

$$b) y = \frac{u^r + 0}{\sqrt{u^r + \varepsilon}} \rightarrow \frac{u^r + \varepsilon}{\sqrt{u^r + \varepsilon}} + \frac{1}{\sqrt{u^r + \varepsilon}}$$

$$\hookrightarrow \sqrt{u^r + \varepsilon} + \frac{1}{\sqrt{u^r + \varepsilon}}$$

②

$$\text{if } u \rightarrow 0, \quad \mu + \frac{1}{\mu} \rightarrow \frac{0}{\mu} \quad \boxed{R_f = \left[\frac{0}{\mu}, +\infty \right)}$$

$$y = |u - \mu| + |\mu u + \mu|$$

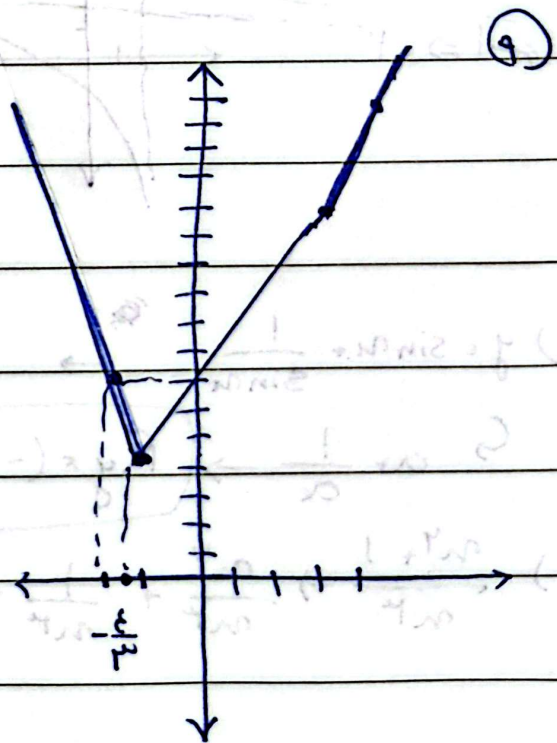
$$\hookrightarrow u < \mu \quad -\frac{\varepsilon}{\mu} \quad \mu$$

$$-u + \mu - \mu u - \varepsilon \quad | \quad -u + \mu + \mu u + \varepsilon \quad | \quad u - \mu + \mu u + \varepsilon$$

$$-u - 1 \quad \left(\frac{1}{\mu} \right) \quad \mu u + \nu \quad \left(\frac{1}{\mu} \right) \quad \varepsilon u + 1$$

$$-\mu, \nu \quad \mu, \nu$$

$$-\mu \rightarrow 11 \quad 0 \rightarrow \mu$$

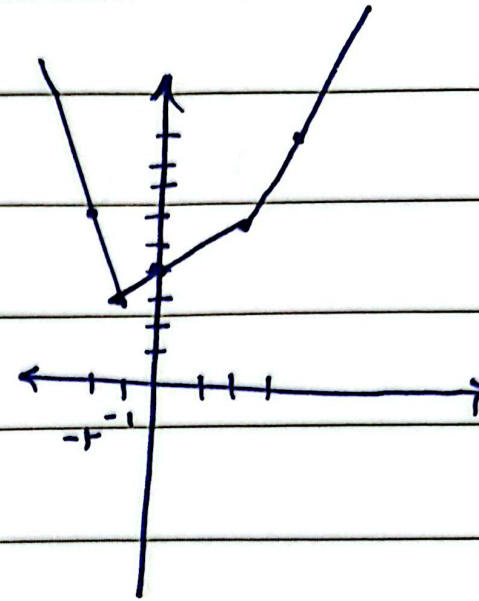


$$y = |x-2| + |x+2|$$

$$Ry = [2, +\infty)$$

$$\begin{array}{c} -1 \quad 2 \\ \hline -x+2 \quad -x+2 \quad x-2+x+2 \\ -x+2 \quad (x) \quad x+2 \quad (x) \quad x+2 \end{array}$$

$$\begin{array}{c|c|c|c|c} -2 & 2 & -1 & 0 & 2 \\ \hline \epsilon & \epsilon & \mu & \epsilon & \mu \end{array}$$



$$y = |x-2| - |x+1|$$

$x=2 \quad x=-1$

$$\begin{array}{c} -1 \quad 1 \\ \hline -x+2 \quad -x+1 \quad x-2 \\ (x) \quad (-2) \end{array} \quad Ry = [-2, +\infty)$$

$$\begin{array}{c|c|c|c|c} -1 & 1 & 0 & 2 & 2 \\ \hline \epsilon & -\epsilon & 1 & -1 & 0 \end{array}$$

