Date   No

1. الف) $y = x^3 - 3x^2 + 3x + 1 - 1 = (x-1)^3 + 1 \Rightarrow (x-1)^3 = y-1 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow R_y = \mathbb{R}$

ب) $y = \dfrac{1}{x^2 - 2x} \rightarrow yx^2 - 2xy = 1 \Rightarrow yx^2 - 2xy - 1 = 0 \Rightarrow x = \dfrac{+2y \pm \sqrt{4y^2 + 4y}}{2y}$

$\begin{array}{c|ccc} & -1 & & 0 \\ \hline & + & - & + \end{array}$

$\Rightarrow y \neq 0 , \quad 4y^2 + 4y \geq 0 \Rightarrow 4y(y+1) \geq 0 \Rightarrow R_y = (-\infty, -1] \cup (0, +\infty)$

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2. الف) $y = x^2 - 4x + 10 \rightarrow x_{min} = \frac{-4}{-2} = 2 \Rightarrow y_{min} = 6 \Rightarrow R_y = [6, +\infty)$

ب) $y = -x^2 + 6x + 3 \rightarrow x_{max} = \frac{-6}{-2} = 3 \Rightarrow y_{max} = 12 \Rightarrow R_y = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3} \rightarrow x_{min} = -\frac{-4}{2} = 2 \Rightarrow y_{min} = -7 \rightarrow [-7, +\infty) \xrightarrow{\sqrt{\ }} R_y = [0, +\infty)$

د) $y = \sqrt{6x - x^2} \rightarrow x_{max} = \frac{-6}{-2} = 3 \Rightarrow y_{max} = 9 \rightarrow (-\infty, 9] \xrightarrow{\sqrt{\ }} R_y = [0, 3]$

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3. الف) $y = x^3 - 5x^2 + 2x + 1 \Rightarrow R_f = \mathbb{R}$

ب) $y = x^7 - 6x^4 + 2x + 2 \Rightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{x^7 - 6x^2 + 5x + 1} \Rightarrow R_f = [0, +\infty)$

د) $y = (x^5 - 4x^2 + 3x + 1)^2 \Rightarrow R_f = (\mathbb{R})^2 \Rightarrow R_f = [0, +\infty)$

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4. الف) $y = \dfrac{3x+1}{x-2} \Rightarrow R_f = \mathbb{R} - \{3\}$

ب) $y = \dfrac{2x+1}{x+1} \Rightarrow R_f = \mathbb{R} - \{2\}$

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5. الف) $y = \sqrt{\dfrac{2x+4}{x+1}} \Rightarrow R_f = [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\dfrac{3x+1}{2-x}} \Rightarrow R_f = [0, +\infty)$

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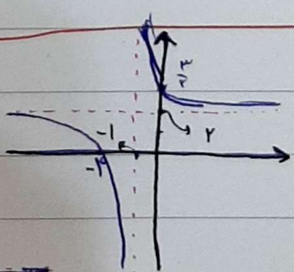
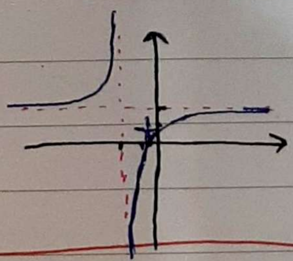
6. الف) $y = \dfrac{2x-3}{x+1} \rightarrow \begin{cases} \text{مجانب قائم } x = -1 \\ \text{مجانب افقی } y = 2 \end{cases}$



ب) $y = \dfrac{2x-1}{x+2} \rightarrow \begin{cases} \text{مجانب قائم } x = -2 \\ \text{مجانب افقی } y = 2 \end{cases}$



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7. الف) $y = \sin x + \dfrac{1}{\sin x} \quad -1 \leq \sin x \leq 1 \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \dfrac{x^6 + 1}{x^3} = x^3 + \dfrac{1}{x^3} \Rightarrow R_f = (-\infty, -2] \cup [2, +\infty)$

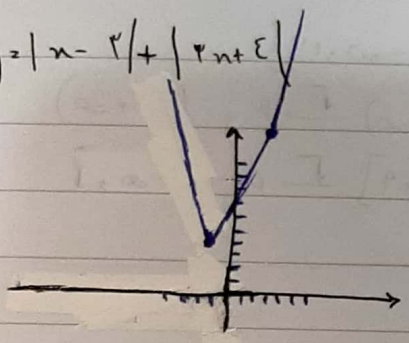
7. ج) $y=\frac{\sqrt[4]{x^2+1}}{\sqrt[4]{x}}=\sqrt[4]{x}+\frac{1}{\sqrt[4]{x}} \longrightarrow R_f=(-\infty,-2]\cup[2,+\infty)$

د) $y=\sqrt{x}+\frac{1}{\sqrt{x}} \Rightarrow R_f=[2,+\infty)$

8. الف) $y=x^2+\frac{1}{x^2+2}+x^2-2=(x^2+2)+\frac{1}{(x^2+2)}-2 \Rightarrow x^2+2 \xrightarrow{\text{کمترین مقدار}} 2 \Rightarrow R_f=[\frac{1}{2},+\infty)$

ب) $y=\frac{x^2+5}{\sqrt{x^2+3}}=\frac{x^2+3+1}{\sqrt{x^2+3}}=\sqrt{x^2+3}+\frac{1}{\sqrt{x^2+3}} \xrightarrow{\text{کمترین مقدار}} 2+\frac{1}{2}=\frac{5}{2} \Rightarrow R_f=[\frac{5}{2},+\infty)$

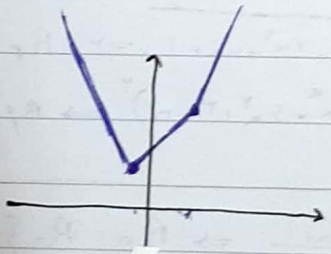
9. $y=|x-2|+|2x+3|$



x	$-\frac{3}{2}$		2	
$-x+2-2x-3$		$x+2+2x+3$		$x-2-2x-3$
$-3x-1$		$3x+5$		$x+3$

10. الف) $y=|x-2|+|2x+2|$

$$\begin{cases} x+2+2x+2 & x\geq 2 \\ -x+2+2x+2 & -1\leq x\leq 2 \\ -x+2-2x-2 & x\leq -1 \end{cases} \Rightarrow \begin{cases} 3x & x>2 \\ x+3 & -1\leq x\leq 2 \\ -3x & x<-1 \end{cases}$$



ب) $y=|2x-2|-|x+1|$

$$\begin{cases} 2x-2-x-1 & x\geq 1 \\ -2x+2-x-1 & -1\leq x\leq 1 \\ -2x-2+x+1 & x\leq -1 \end{cases} \Rightarrow \begin{cases} x-3 & x\geq 1 \\ -3x+1 & -1\leq x\leq 1 \\ -x-3 & x\leq -1 \end{cases}$$