

1. a) $y = x^3 - 3x^2 + 3x + 1 - 1 = (x-1)^3 + 1 \Rightarrow (x-1)^3 = y-1 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow R_y = \mathbb{R}$

b) $y = \frac{1}{x^2 - 2x} \rightarrow yx^2 - 2xy = 1 \Rightarrow yx^2 - 2xy - 1 = 0 \Rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$

$\Rightarrow y \neq 0, \quad 4y^2 + 4y \geq 0 \Rightarrow 4y(y+1) \geq 0 \Rightarrow R_y = (-\infty, -1] \cup (0, +\infty)$

2. a) $y \geq x^2 - 8x + 10 \rightarrow x_{\min} = \frac{8}{2} = 4 \Rightarrow y_{\min} = 4 \Rightarrow R_y = [4, +\infty)$

b) $y = -x^2 + 4x + 3 \rightarrow x_{\max} = \frac{4}{2} = 2 \Rightarrow y_{\max} = 11 \Rightarrow R_y = (-\infty, 11]$

c) $y = \sqrt{x^2 - 8x - 3} \rightarrow x_{\min} = \frac{8}{2} = 4 \Rightarrow y_{\min} = -1 \rightarrow [-1, +\infty) \xrightarrow{\sqrt{\cdot}} R_y = [0, +\infty)$

d) $y = \sqrt{7x - x^2} \rightarrow x_{\max} = \frac{7}{2} = 3.5 \Rightarrow y_{\max} = 9 \rightarrow [-\infty, 9] \xrightarrow{\sqrt{\cdot}} R_y = [0, 3]$

3. a) $y = x^2 - 2x + 1 \Rightarrow R_f = \mathbb{R}$

b) $y = x^2 - 8x + 10 \Rightarrow R_f = \mathbb{R}$

c) $y = \sqrt{x^2 - 4x + 2x + 1} \Rightarrow R_f = [0, +\infty)$

d) $y = (x^2 - 8x + 10)^2 \Rightarrow R_f = (\mathbb{R})^2 \Rightarrow R_f = [0, +\infty)$

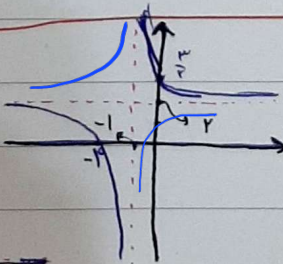
4. a) $y = \frac{x+1}{x-1} \Rightarrow R_f = \mathbb{R} - \{1\}$

b) $y = \frac{x+1}{x+1} \Rightarrow R_f = \mathbb{R} - \{1\}$

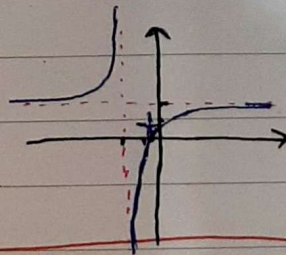
5. a) $y = \sqrt{\frac{x+1}{x+1}} \Rightarrow R_f = [0, +\infty) - \{1\}$

b) $y = \sqrt{\frac{x+1}{x-1}} \Rightarrow R_f = [0, +\infty)$

6. a) $y = \frac{x-1}{x+1} \rightarrow \begin{cases} \text{Asymptote } x = -1 \\ \text{Asymptote } y = 1 \end{cases}$



b) $y = \frac{x-1}{x+1} \rightarrow \begin{cases} \text{Asymptote } x = -1 \\ \text{Asymptote } y = -1 \end{cases}$



7. a) $y = \sin x + \frac{1}{\sin x} \rightarrow -1 \leq \sin x \leq 1 \Rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

b) $y = \frac{x+1}{x^2} = x^2 + \frac{1}{x^2} \Rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

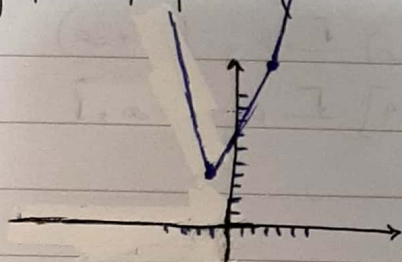
7. ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow R_f = [1, +\infty)$

8. الف) $y = x^2 + \frac{1}{x^2+3} + 3-3 = (x^2+3) + \frac{1}{(x^2+3)} - 3 \xrightarrow{\text{تجزیه}} x^2+3 \geq 3 \Rightarrow R_f = [\frac{1}{3}, +\infty)$ (5)

ب) $y = \frac{x^2+a}{\sqrt{x^2+\varepsilon}} = \frac{x^2+\varepsilon+1}{\sqrt{x^2+\varepsilon}} = \sqrt{x^2+\varepsilon} + \frac{1}{\sqrt{x^2+\varepsilon}} \xrightarrow{\text{تجزیه}} 2 + \frac{1}{\sqrt{\varepsilon}} = \frac{a}{\sqrt{\varepsilon}} \Rightarrow R_f = [\frac{a}{\sqrt{\varepsilon}}, +\infty)$

9. $y = |x-2| + |2x+\varepsilon|$



$-x+2-x-\varepsilon$ $-\varepsilon x-1$	$-x+2+2x+\varepsilon$ $x+2$	$x-2+2x+\varepsilon$ $3x-\varepsilon+1$
$-\frac{\varepsilon}{3}$	$\frac{1}{3}$	2

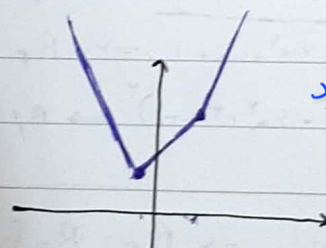
(2)

10. الف) $y = |x-2| + |2x+2|$

$$\begin{cases} x-2+2x+2 & x \geq 2 \\ -x+2+2x+2 & -1 \leq x \leq 2 \\ -x+2-2x-2 & x \leq -1 \end{cases} \Rightarrow \begin{cases} 3x & x \geq 2 \\ x+4 & -1 \leq x \leq 2 \\ -3x & x \leq -1 \end{cases}$$

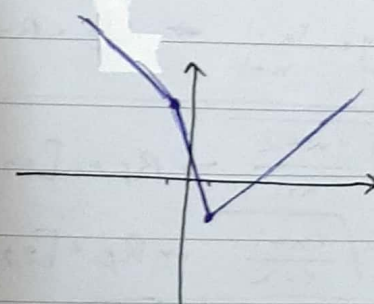
ب) $y = |2x-2| - |x+1|$

$$\begin{cases} 2x-2-x-1 & x \geq 1 \\ -2x+2-x-1 & -1 \leq x \leq 1 \\ -2x+2+x+1 & x \leq -1 \end{cases} \Rightarrow \begin{cases} x-3 & x \geq 1 \\ -3x+1 & -1 \leq x \leq 1 \\ -x+3 & x \leq -1 \end{cases}$$



نقطه بد

$R = [-1, +\infty)$



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