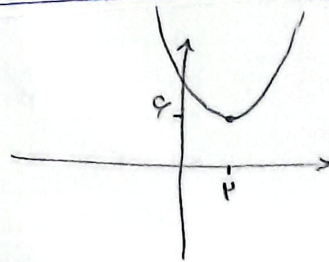


الف) $x^3 - 3x^2 + 3x = y \rightarrow x^3 - 3x^2 + 3x - 1 + 1 = y \rightarrow (x-1)^3 = y-1 \rightarrow x = \sqrt[3]{y-1} + 1$
 $R_f = \mathbb{R}$

ب) $\frac{1}{x^2 - 2x} = y \rightarrow yx^2 - 2xy = 1 \rightarrow yx^2 - 2xy - 1 = 0 \rightarrow \frac{-b \pm \sqrt{\Delta}}{2a} \rightarrow \frac{2y \pm \sqrt{\Delta}}{2y}$
 $y \neq 0$
 $\Delta \geq 0 \rightarrow b^2 - 4ac \geq 0 \rightarrow 4y^2 + 4y \geq 0$
 $4y(y+1) \geq 0 \rightarrow \frac{-1}{+|-|+}$
 $R_f = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 1 \rightarrow x = \frac{-b}{2a} = \frac{4}{2} = 2 \rightarrow y = 4 - 8 + 1 = -3 \rightarrow$



ب) $y = -x^2 + 4x + 3 \rightarrow \frac{-b}{2a} = \frac{-4}{-2} = 2 \rightarrow -4 + 8 + 3 = 7 = y$
 $R_f = [7, +\infty)$

ج) $y = \sqrt{x^2 - 4x - 3} \rightarrow \frac{-b}{2a} = \frac{4}{2} = 2 \rightarrow 4 - 8 - 3 = -7 \rightarrow [0, +\infty) = R_f$

د) $y = \sqrt{4x - x^2} \rightarrow \frac{-b}{2a} = \frac{-4}{-2} = 2 \rightarrow y = 8 - 4 = 4 \rightarrow [0, 4] = R_f$

الف) $R_f = \mathbb{R}$ د) $[0, +\infty)$

ب) $R_f = \mathbb{R}$

ج) $R_f = [0, +\infty)$

الف) $R_f = \mathbb{R} - \{3\}$ ب) $R_f = \mathbb{R} - \{2\}$

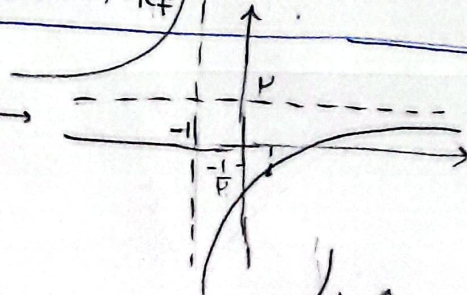
الف) $y = \sqrt{\frac{4x+4}{x+1}} \rightarrow \frac{4}{4} \rightarrow [0, +\infty) - \{2\} = R_f$

ب) $y = \sqrt{\frac{4x+1}{x-1}} \rightarrow \dots \rightarrow [0, +\infty) = R_f$

الف) $y = \frac{4x-3}{x+1} \rightarrow x = -1$

$x = 1 \rightarrow y = \frac{4-3}{1+1} = \frac{1}{2}$

$y = 2$

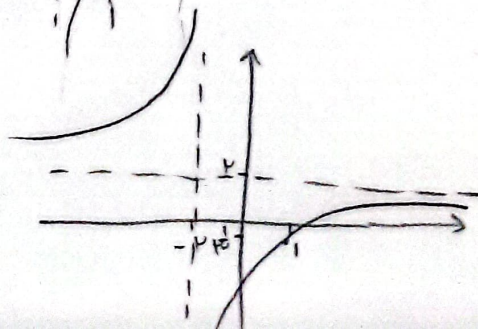


$\mathbb{R} - \{2\} = R_f$

ب) $\frac{4x-1}{x+2} = y \rightarrow x = -2$

$x = 1 \rightarrow \frac{4-1}{1+2} = \frac{1}{3}$

$y = 2$



$\mathbb{R} - \{2\} = R_f$

الف) $y = \sin x + \frac{1}{\sin x} \rightarrow (-\infty, -1] \cup [1, +\infty) = \mathbb{R}_f$

7

ب) $\frac{x^p+1}{x^{p+1}} = y \rightarrow x^p + \frac{1}{x^p} = y \rightarrow (-\infty, -1] \cup [1, +\infty) = \mathbb{R}_f$

ج) $\sqrt[p]{x} + \frac{1}{\sqrt[p]{x}} = y \rightarrow (-\infty, -1] \cup [1, +\infty) = \mathbb{R}_f$

د) $[1, +\infty) = \mathbb{R}_f$

الف) $x^p + \frac{1}{x^{p+1}} - \frac{1}{x^p} = y \rightarrow x \rightarrow y = \frac{1}{x^p} \rightarrow [\frac{1}{x^p}, +\infty) = \mathbb{R}_f$

8

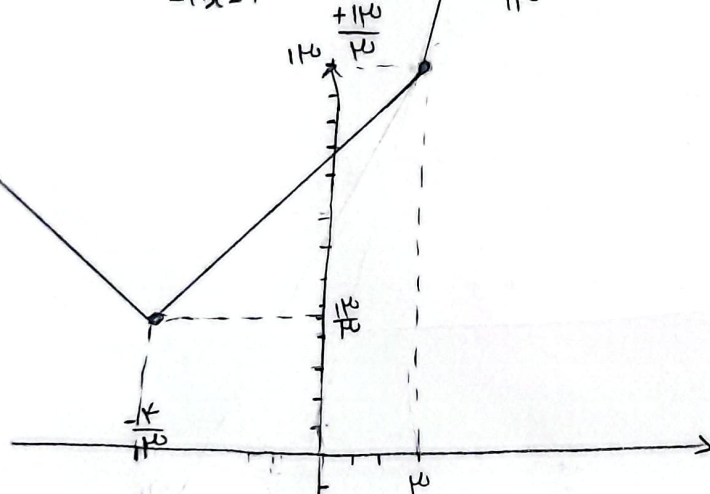
ب) $\frac{x^{p+1}+1}{\sqrt{x^{p+1}}} \rightarrow \sqrt{x^{p+1}} + \frac{1}{\sqrt{x^{p+1}}} = y \rightarrow x \rightarrow y = \sqrt{x} + \frac{1}{\sqrt{x}} = \frac{x}{\sqrt{x}} \rightarrow [\frac{x}{\sqrt{x}}, +\infty) = \mathbb{R}_f$

9

$y = |x-p| + |x+q| \rightarrow$

$\frac{-p}{p} - p = \frac{-1p}{p}$

$p-x-p-x = \frac{p-x+p-x}{p-x+p} \frac{p-x+1}{1p}$



$\mathbb{R}_f = [\frac{1p}{p}, +\infty)$

الف) $-1 \quad p$
 $p-x-p-x-p = \frac{p-x+p-x+p}{x+p} \frac{p-x}{p-x}$
 $-p-x$
 $+p$

$\mathbb{R}_f = [p, +\infty)$

10

ب) $-1 \quad 1$
 $+x+1-p-x+p = \frac{p-p-x-x-1}{-p-x+1} \frac{x-p}{x-p}$
 $-x+p$
 $-p$

$\mathbb{R}_f = [-p, +\infty)$

