

1- الف) $y = x^3 - 3x^2 + 3x - 1 + 1 \rightarrow y = (x-1)^3 + 1 \rightarrow \sqrt[3]{y-1} + 1 = x \Rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 1} \rightarrow x^2 y - 2xy + y = 1 + y \rightarrow y(x-1)^2 = 1+y \rightarrow \sqrt[2]{\frac{1+y}{y}} + 1 = x$

$\rightarrow \frac{1+y}{y} \geq 0 \quad \frac{-1}{+1-1} \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

2- الف) $y = x^2 - 4x + 1 \rightarrow \min \left| \begin{array}{l} \frac{y}{x} = 2 \\ x - 1 + 1 = 0 \end{array} \right| \quad [0, +\infty) = R_f$

ب) $y = -x^2 + 4x + 3 \rightarrow \max \left| \begin{array}{l} \frac{y}{x} = 2 \\ -x + 4 + 3 = 0 \end{array} \right| \rightarrow R_f = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3} \Rightarrow \min \left| \begin{array}{l} \frac{y}{x} = 2 \\ x - 1 - 3 = 0 \end{array} \right| \Rightarrow R_f = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \Rightarrow \max \left| \begin{array}{l} \frac{y}{x} = 2 \\ 4 - x = 0 \end{array} \right| \Rightarrow R_f = [0, 3]$

3- الف) \mathbb{R} ب) \mathbb{R} ج) $[0, +\infty)$ د) $[0, +\infty)$

4- الف) $y = \frac{x+1}{x-1} \rightarrow \mathbb{R} - \{1\} = R_f$

ب) $y = \frac{x+1}{x+1} \rightarrow \mathbb{R} - \{1\} = R_f$

5- الف) $y = \sqrt{\frac{x+4}{x+1}} \rightarrow R_f = [0, +\infty) - \{1\}$

ب) $y = \sqrt{\frac{x+1}{x-1}} \rightarrow R_f = [0, +\infty)$

6- الف) $y = \frac{x-3}{x+1}$

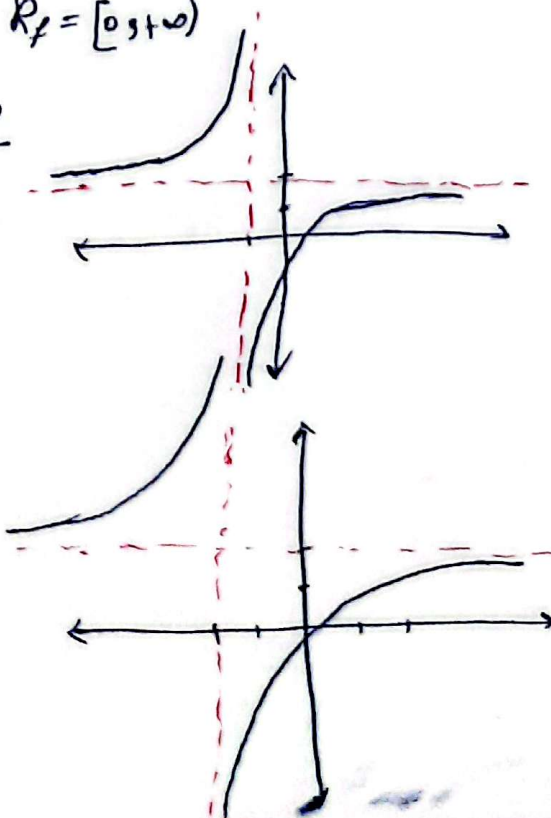
مجانبات افقي $\Rightarrow y = 2$
مجانبات عمودي $\Rightarrow x = -1$

$\begin{vmatrix} 0 \\ -3 \end{vmatrix}$

ب) $y = \frac{x-1}{x+2}$

مجانبات افقي $\Rightarrow y = 2$
مجانبات عمودي $\Rightarrow x = -2$

$\begin{vmatrix} 0 \\ -1/2 \end{vmatrix}$



V-

الف) $y = \sin x + \frac{1}{\sin x} \rightarrow (-\infty, -1] \cup [1, +\infty) = R_f$

ب) $y = x^p + \frac{1}{x^p} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

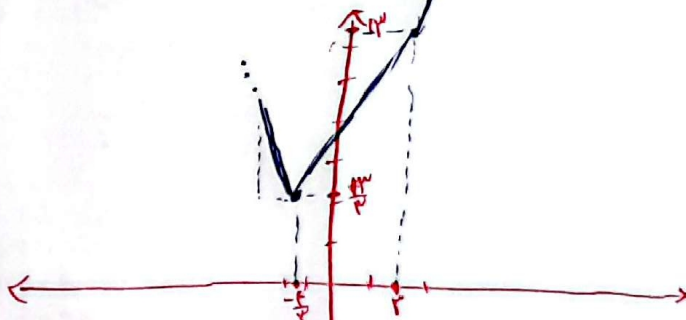
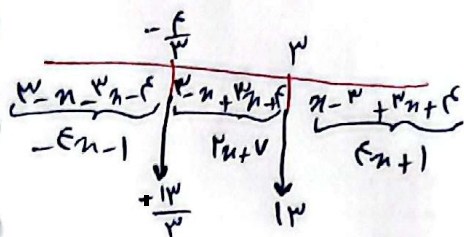
ج) $y = \sqrt[n]{x} + \frac{1}{\sqrt[n]{x}} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

د) $\sqrt{x} + \frac{1}{\sqrt{x}} = y \rightarrow R_f = [1, +\infty)$

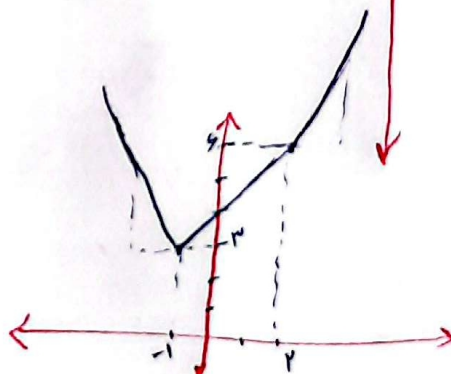
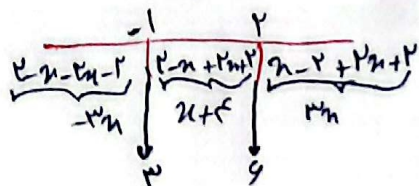
الف) $y = \underbrace{x^p + \frac{1}{x^p}}_{\downarrow} - p \Rightarrow R_f = [\frac{1}{p}, +\infty)$

ب) $y = \frac{x^p + c}{\sqrt{x^p + c}} = \sqrt{x^p + c} + \frac{1}{\sqrt{x^p + c}} \Rightarrow R_f = [\frac{c}{p}, +\infty)$

9- $y = |x - p| + |x + p|$

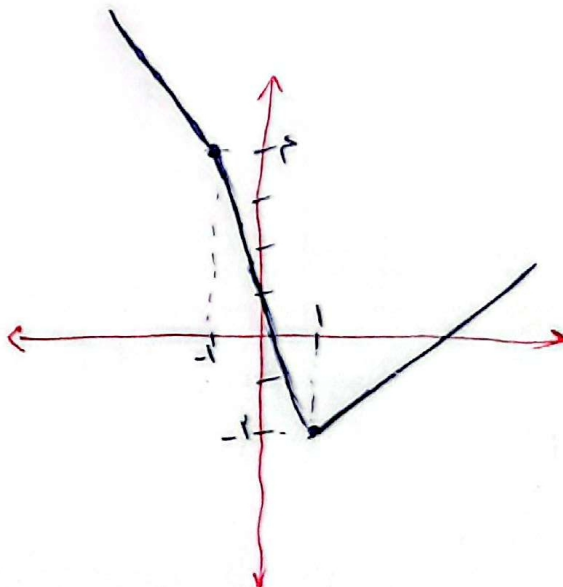
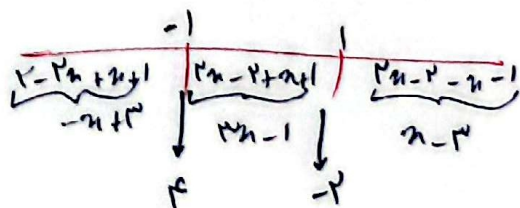


الف) $y = |x - p| + |x + p|$



$R_f = [2p, +\infty)$

ب) $y = |x - p| - |x + 1|$



$R_f = [-p - 1, p + 1]$