

۱- الف) $y = x^3 - 3x^2 + 3x - 1 + 1 \rightarrow y = (x-1)^3 + 1 \rightarrow \sqrt[3]{y-1} + 1 = x \Rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \rightarrow x^2y - 2xy + y = 1 + y \rightarrow y(x-1)^2 = 1+y \rightarrow \sqrt[2]{\frac{1+y}{y}} + 1 = x$

$\rightarrow \frac{1+y}{y} \geq 0 \rightarrow \frac{-1}{+1-2} \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$

۲- الف) $y = x^2 - 4x + 1 \rightarrow \min \left| \begin{array}{l} \frac{f}{x} = 2 \\ f - 4x + 1 = 0 \end{array} \right| \Rightarrow [0, +\infty) = R_f$

ب) $y = -x^2 + 4x + 3 \rightarrow \max \left| \begin{array}{l} \frac{f}{x} = 2 \\ -x^2 + 4x + 3 = 0 \end{array} \right| \Rightarrow R_f = (-\infty, 11]$

ج) $y = \sqrt{x^2 - 4x - 3} \Rightarrow \min \left| \begin{array}{l} \frac{f}{x} = 2 \\ x^2 - 4x - 3 = 0 \end{array} \right| \Rightarrow R_f = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \Rightarrow \max \left| \begin{array}{l} \frac{f}{x} = 2 \\ 4x - x^2 = 0 \end{array} \right| \Rightarrow R_f = [0, 4]$

۳- الف) $\mathbb{R}$ ب) $\mathbb{R}$ ج) $[0, +\infty)$ د) $[0, +\infty)$

۴- الف) $y = \frac{x+1}{x-1} \rightarrow \mathbb{R} - \{1\} = R_f$

ب) $y = \frac{x+1}{x+1} \rightarrow \mathbb{R} - \{1\} = R_f$

۵- الف) $y = \sqrt{\frac{x+1}{x+1}} \rightarrow R_f = [0, +\infty) - \{1\}$

ب) $y = \sqrt{\frac{x+1}{x-1}} \rightarrow R_f = [0, +\infty)$

۶- الف) $y = \frac{x-1}{x+1}$

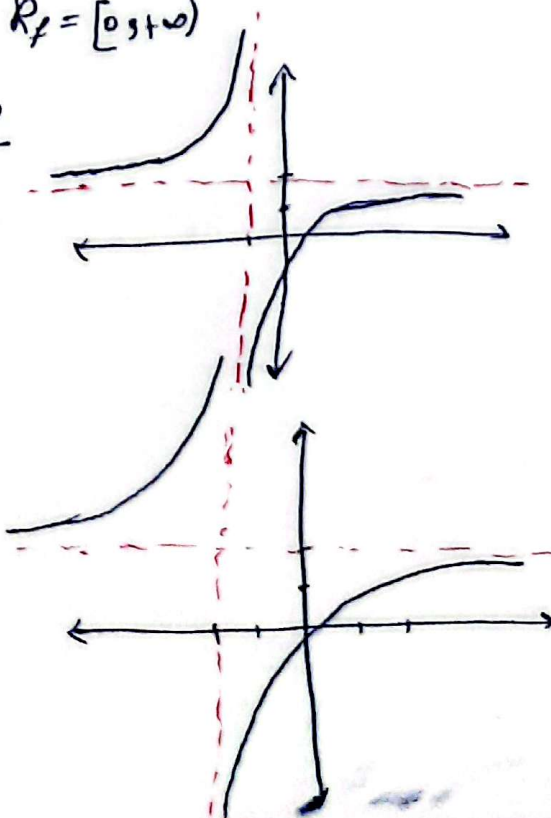
میان افقی $\Rightarrow y = 1$
میان عمودی $\Rightarrow x = -1$

$\begin{matrix} 0 \\ -1 \end{matrix}$

ب) $y = \frac{x-1}{x+1}$

میان افقی $\Rightarrow y = 1$
میان عمودی $\Rightarrow x = -1$

$\begin{matrix} 0 \\ -1 \end{matrix}$



6- الف) $y = \sin x + \frac{1}{\sin x} \rightarrow (-\infty, -1] \cup [1, +\infty) = R_f$

ب) $y = x^p + \frac{1}{x^p} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

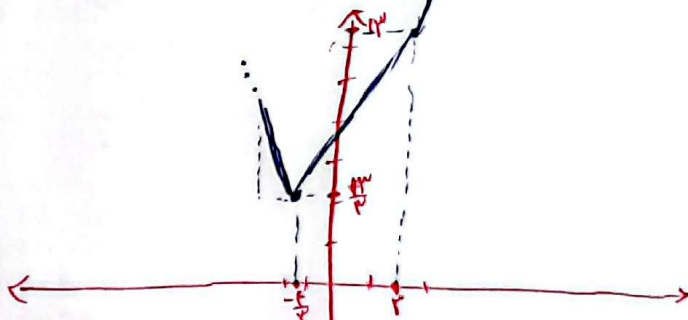
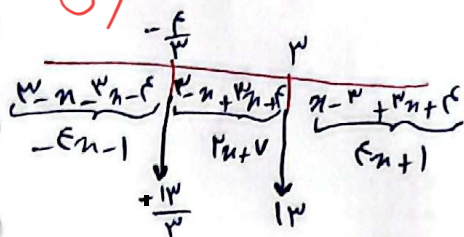
ج) $y = \sqrt[n]{x} + \frac{1}{\sqrt[n]{x}} \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

د) $\sqrt{x} + \frac{1}{\sqrt{x}} = y \rightarrow R_f = [1, +\infty)$

7- الف) $y = \underbrace{x^p + \frac{1}{x^p}}_{[\frac{1}{p}, +\infty)} - p \Rightarrow R_f = [\frac{1}{p}, +\infty)$

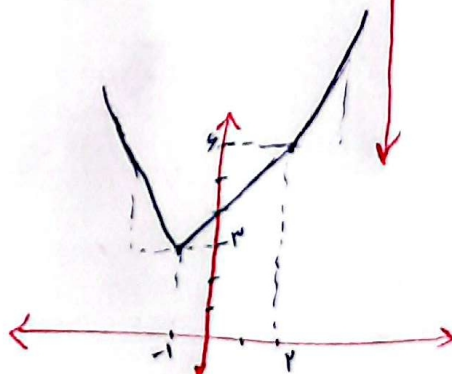
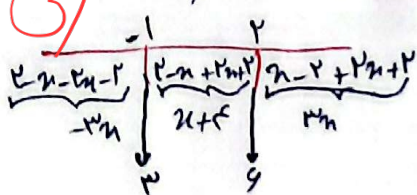
ب) $y = \frac{x^p + c}{\sqrt{x^p + c}} = \sqrt{x^p + c} + \frac{1}{\sqrt{x^p + c}} \Rightarrow R_f = [\frac{c}{p}, +\infty)$

8- $y = |x - p| + |x + p|$

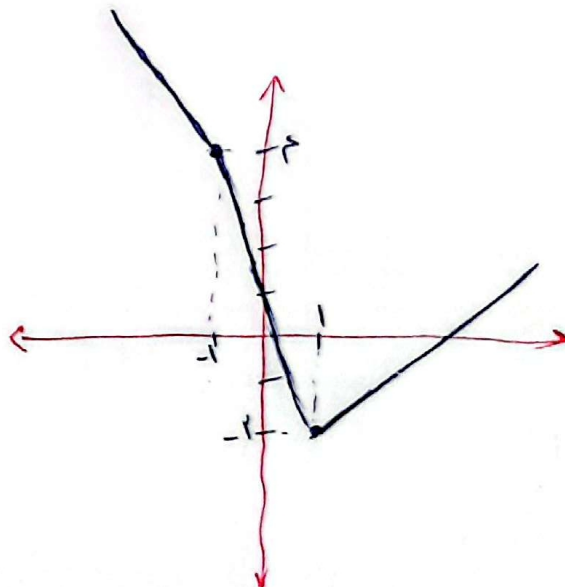
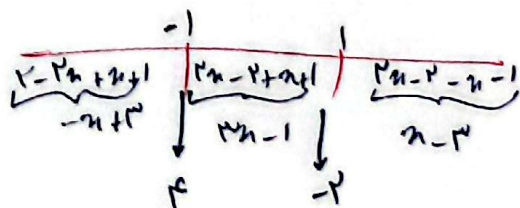


$R_f = [2p, +\infty)$

9- الف) $y = |x - p| + |x + p|$



ب) $y = |x - p| - |x + 1|$



$R_f = [-p - 1, +\infty)$