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الف) $y = x^3 - 3x^2 + 3x \rightarrow (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow x = \sqrt[3]{y-1} + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 1} \rightarrow yx^2 - 2xy - 1 = 0 \rightarrow \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 2\sqrt{y(y+1)} \rightarrow \frac{-1 \pm \sqrt{1+4y}}{2y} + 1$
 $R_f = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 1 \rightarrow y = \frac{-\Delta}{2a} = \frac{-(16 - 4 \cdot 1 \cdot 1)}{2 \cdot 1} = 7 \rightarrow R = [7, +\infty)$

ب) $y = -x^2 + 4x + 2 \rightarrow y = \frac{-(16 - 4(-1)(2))}{-2} = 12 \rightarrow R = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 2} \rightarrow y = \frac{-(16 - 4 \cdot 1 \cdot (-2))}{2} = -10 \rightarrow R = \sqrt{[-10, +\infty)} = [0, +\infty)$

د) $y = \sqrt{-x^2 + 4x} \rightarrow y = \frac{-(16 - 4(-1)(0))}{-2} = 9 \rightarrow R = \sqrt{(-9, +\infty)} = [0, 3]$

الف) $y = x^3 - 6x^2 + 9x + 1 \rightarrow R = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 3x + 2 \rightarrow R = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 6x + 1} \rightarrow R = [0, +\infty)$

د) $y = (x^3 - 4x^2 + 3x + 1)^2 \rightarrow R = [0, +\infty)$

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الف) $y = \frac{3x+1}{x-2} \rightarrow R = \mathbb{R} - \{3\}$

ب) $y = \frac{2x+1}{x+1} \rightarrow R = \mathbb{R} - \{1\}$

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الف) $y = \sqrt{\frac{2x+4}{x+1}} \rightarrow R = \sqrt{\mathbb{R} - \{1\}} \rightarrow R = [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\frac{2x+1}{x}} \rightarrow y = \sqrt{\frac{2x+1}{-x+1}} \rightarrow R = \sqrt{\mathbb{R} - \{1\}} \rightarrow R = [0, +\infty)$

الف) $y = \frac{2x - 3}{x + 1}$

→ -1 جانب عمودي

→ 2 جانب افقي

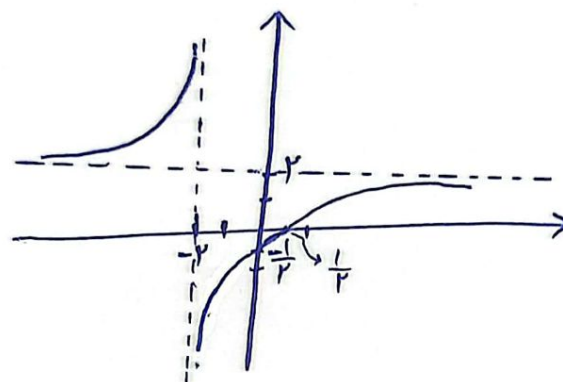
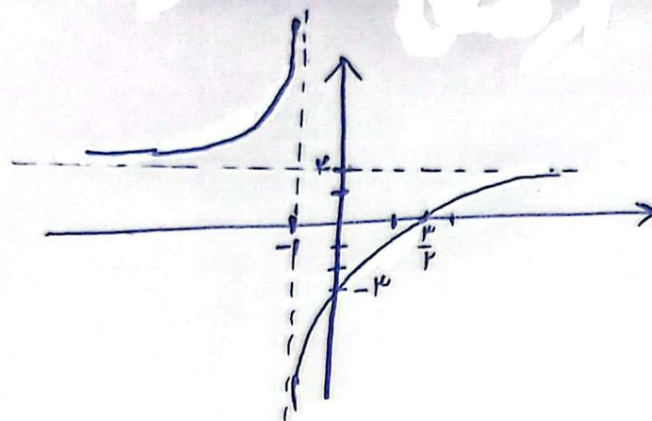
$$\frac{2}{1} - (-3) = 5 > 0$$

ب) $y = \frac{2x - 1}{x + 2}$

→ -2 جانب عمودي

→ 2 جانب افقي

$$\frac{2}{-1} - (-1) = -1 < 0$$



$$a) y = \sin x + \frac{1}{\sin x} \rightarrow B = (-\frac{\pi}{2}, -1] \cup [1, +\infty)$$

$$b) y = \frac{x^q + 1}{x^p} = x^r + \frac{1}{x^p} \rightarrow B = (-\infty, -1] \cup [1, +\infty)$$

$$c) y = \frac{\sqrt[p]{x^r} + 1}{\sqrt[p]{x}} = \sqrt[p]{x} + \frac{1}{\sqrt[p]{x}} \rightarrow B = (-\infty, -1] \cup [1, +\infty)$$

$$d) y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow B = [1, +\infty)$$

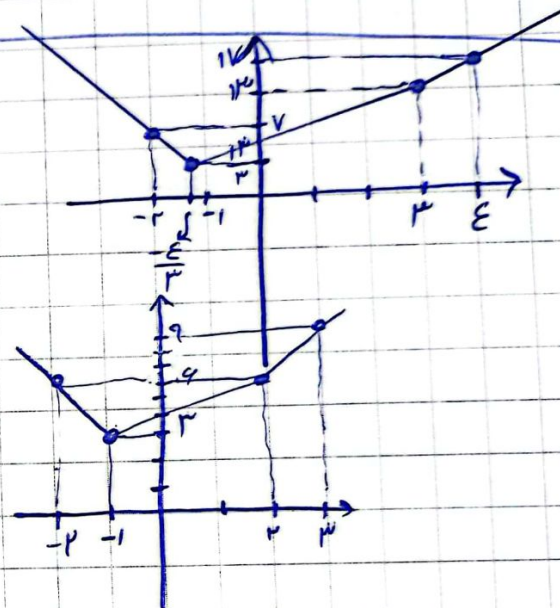
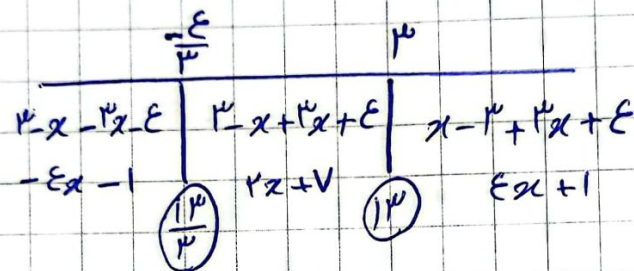
$$e) y = x^r + \frac{1}{x^r + p} \rightarrow \left((x^r + p) + \frac{1}{x^r + p} \right) - p \rightarrow B = [\frac{1}{p}, +\infty)$$

$$f) y = \frac{x^r + q}{\sqrt{x^r + e}} = \sqrt{x^r + e} + \frac{1}{\sqrt{x^r + e}} \rightarrow B = [\frac{0}{1}, +\infty)$$

sometimes
never

$$y = |x - \mu| + |\mu x + \varepsilon|$$

$\downarrow$ $\downarrow$
 $x = \mu$ $x = -\frac{\varepsilon}{\mu}$



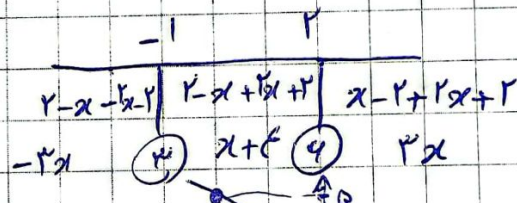
9

10

a) $y = |x - r| + |\mu x + r|$

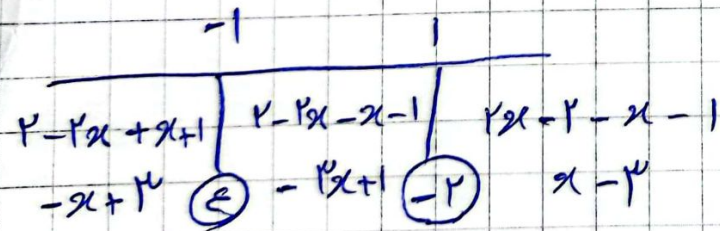
$\downarrow$ $\downarrow$
 $x = r$ $x = -1$

$R = [\mu, +\infty)$



c) $y = |r x - r| - |x + 1|$

$\downarrow$ $\downarrow$
 $x = 1$ $x = -1$



$R = [-r, +\infty)$

