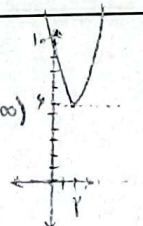


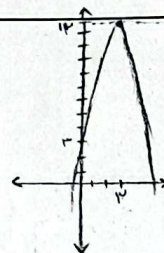
الف) $x^3 - 3x^2 + 3x - 1 + 1 \rightarrow y = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1$
 $\rightarrow x = 1 + \sqrt[3]{y-1} \rightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \rightarrow \frac{1}{y} = (x^2 - 2x + 1) - 1 \rightarrow \frac{1}{y} = (x-1)^2 - 1 \rightarrow \frac{1}{y} + 1 = (x-1)^2$
 $\rightarrow 1 \pm \sqrt{\frac{1+y}{y}} = x \rightarrow \begin{array}{c} -1 \\ | + \quad - \\ 1 \quad 0 \end{array} \quad R_f = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 6$
 $a > 0$
 $\frac{a}{b, \min} \rightarrow \begin{cases} x = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y = 4 - 16 + 6 = -6 \end{cases} \quad R_f = [4, +\infty)$



ب) $y = -x^2 + 4x + 3$
 $a < 0$
 $\frac{a}{b, \max} \rightarrow \begin{cases} x = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y = -9 + 16 + 3 = 10 \end{cases} \quad R_f = (-\infty, 10]$



ج) $y = \sqrt{x^2 - 4x - 3}$
 $a > 0$
 $\frac{a}{b, \min} \rightarrow \begin{cases} x = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y = 4 - 8 - 3 = -7 \end{cases} \quad R_f = [-7, +\infty) = [0, +\infty)$

د) $y = \sqrt{4x - x^2}$
 $a < 0$
 $\frac{a}{b, \max} \rightarrow \begin{cases} x = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y = 16 - 4 = 12 \end{cases} \quad R_f = \sqrt{(-\infty, 12]} = [0, 12]$

الف) $y = x^2 - 5x^2 + 2x + 1$ $\xrightarrow{\text{برای هر } x \in \mathbb{R}}$ $R_f = \mathbb{R}$

ب) $y = x^5 - 4x^4 + 3x + 2$ $\xrightarrow{\text{برای هر } x \in \mathbb{R}}$ $R_f = \mathbb{R}$

ج) $y = \sqrt{x^4 - 4x^2 + 5x + 1}$ $\xrightarrow{\text{برای هر } x \in \mathbb{R}}$ $R_f = \sqrt{\mathbb{R}} : [0, +\infty)$

د) $y = (x^5 - 4x^4 + 3x + 1)^2$ $\xrightarrow{\text{برای هر } x \in \mathbb{R}}$ $R_f = [0, +\infty)$

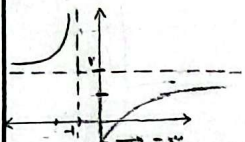
الف) $y = \frac{cx+1}{x-2}$ $\frac{ad \neq bc, c \neq 0}{-4 \neq 1, 1 \neq 0} \rightarrow R_f : \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_f : \mathbb{R} - \{2\}$

ب) $y = \frac{cx+1}{x+1}$ $\frac{ad \neq bc, c \neq 0}{1 \neq 1, 1 \neq 0} \rightarrow R_f : \mathbb{R} - \left\{\frac{a}{c}\right\} \rightarrow R_f : \mathbb{R} - \{1\}$

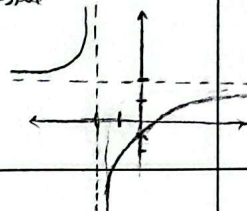
الف) $y = \sqrt{\frac{cx+1}{x+1}}$ $\frac{ad \neq bc, c \neq 0}{1 \neq 1, 1 \neq 0} \rightarrow R_f : \sqrt{\mathbb{R} - \{1\}} \rightarrow R_f : [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\frac{cx+1}{-x+1}}$ $\frac{ad \neq bc, c \neq 0}{1 \neq -1, 1 \neq 0} \rightarrow R_f : \sqrt{\mathbb{R} - \{-1\}} \rightarrow R_f : [0, +\infty)$

الف) $y = \frac{2x-3}{x+1}$
 جانب نام (ریشه خارج) $x+1=0 \rightarrow x=-1$
 جانب باقی $\frac{c}{a} = 2 \rightarrow y=2$
 $ad-bc = 2-(-2) = 4 > 0 \rightarrow \text{دایره}$



ب) $y = \frac{2x-1}{x+2}$
 جانب نام (ریشه خارج) $x+2=0 \rightarrow x=-2$
 جانب باقی $\frac{c}{a} = 2 \rightarrow y=2$
 $ad-bc = 4+1 = 5 > 0 \rightarrow \text{دایره}$



الف) $y = \sin x + \frac{1}{\sin x}$ $-1 \leq \sin x \leq 1 \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

ب) $y = \frac{x^4+1}{x^3} = x + \frac{1}{x^3}$ $\frac{1}{x^3} < 0 \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\frac{1}{\sqrt{x}} > 0 \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\frac{1}{\sqrt{x}} > 0 \rightarrow R_f = [1, +\infty)$

الف) $y = (x^2 + \frac{1}{x^2+r} + r) - r = y = (x^2+r + \frac{1}{x^2+r}) - r \xrightarrow{x^2+r > 0} R_f = [\frac{1}{r}, +\infty) - r \rightarrow R_f = [\frac{1}{r}, +\infty)$

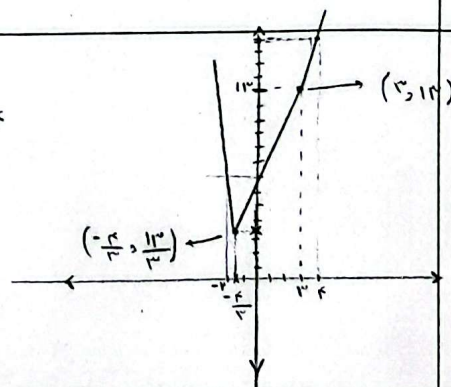
دو عبارت $\alpha + \frac{1}{\alpha}$ در صورتی که α متغیر باشد، مقدار حداقل 2 می شود. در صورتی که α متغیر باشد، مقدار حداقل 10 می شود. $R_f = [10, +\infty)$

ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \frac{x^2+4+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}}$ $\sqrt{x^2+4} > 0 \rightarrow R_f = [\frac{5}{2}, +\infty)$

در صورتی که $x=0 \rightarrow y = \sqrt{4} + \frac{1}{\sqrt{4}} = 2 + \frac{1}{2} = \frac{5}{2}$

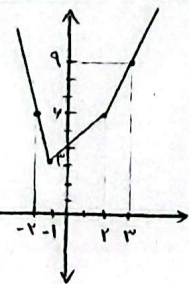
$y = |x-3| + |3x+4|$
 در صورتی که $x=3$ $\frac{-4}{3}$ $\frac{3}{3}$
 $-x+3-3x = -4x+6$ $\frac{-4}{3}$ $\frac{3}{3}$
 $= -4x+6$ $\frac{-4}{3}$ $\frac{3}{3}$
 $= -4x+6$ $\frac{-4}{3}$ $\frac{3}{3}$

$\begin{cases} x=-2 \rightarrow y=5 \\ x=-\frac{4}{3} \rightarrow y=\frac{10}{3} \\ x=3 \rightarrow y=10 \\ x=4 \rightarrow y=14 \end{cases}$



الف) $y = \frac{|x-2|}{x+2} + \frac{|2x+1|}{x-1}$

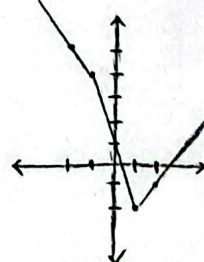
$\begin{cases} x=-2 \rightarrow y=4-1=3 \\ x=-1 \rightarrow y=3 \\ x=2 \rightarrow y=4 \\ x=3 \rightarrow y=1+8=9 \end{cases}$



$R_f = [3, +\infty)$

ب) $y = \frac{|2x-2|}{x+1} - \frac{|x+1|}{x-1}$

$\begin{cases} x=-2 \rightarrow y=4-1=3 \\ x=-1 \rightarrow y=0 \\ x=1 \rightarrow y=-2 \\ x=2 \rightarrow y=2-3=-1 \end{cases}$



$R_f = [-1, +\infty)$