

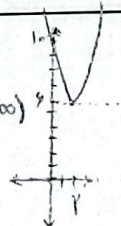
19,5

الف) $x^3 - 3x^2 + 3x - 1 + 1 \rightarrow y = (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1$
 $\rightarrow x = 1 + \sqrt[3]{y-1} \rightarrow R_f: \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \rightarrow \frac{1}{y} = (x^2 - 2x + 1) - 1 \rightarrow \frac{1}{y} = (x-1)^2 - 1 \rightarrow \frac{1}{y} + 1 = (x-1)^2$
 $\rightarrow 1 \pm \sqrt{\frac{1+y}{y}} = x \rightarrow \begin{matrix} -1 \\ + \end{matrix} \begin{matrix} - \\ + \end{matrix} \begin{matrix} 0 \\ 1 \end{matrix}$ $R_f: (-\infty, -1] \cup (0, +\infty)$

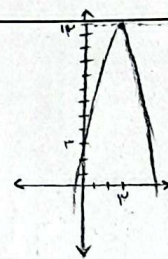
الف) $y = x^2 - 4x + 6$

$a > 0$
 $\frac{a}{b \min}$ $\left| \begin{matrix} x = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y = 4 - 16 + 6 = -6 \end{matrix} \right. \quad R_f: [4, +\infty)$



ب) $y = -x^2 + 4x + 3$

$a < 0$
 $\frac{a}{b \max}$ $\left| \begin{matrix} x = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y = -9 + 16 + 3 = 10 \end{matrix} \right. \quad R_f: (-\infty, 10]$



ج) $y = \sqrt{x^2 - 4x - 3}$

$a > 0$
 $\frac{a}{b \min}$ $\left| \begin{matrix} x = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y = 4 - 8 - 3 = -7 \end{matrix} \right. \quad R_f = [-7, +\infty) = [0, +\infty)$

د) $y = \sqrt{4x - x^2}$

$a < 0$
 $\frac{a}{b \max}$ $\left| \begin{matrix} x = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y = 16 - 4 = 12 \end{matrix} \right. \quad R_f = \sqrt{(-\infty, 9]} = [0, 3]$

الف) $y = x^2 - 5x^2 + 2x + 1$ $R_f: \mathbb{R}$

ب) $y = x^5 - 4x^4 + 3x + 2$ $R_f: \mathbb{R}$

ج) $y = \sqrt{x^4 - 4x^2 + 5x + 1}$ $R_f: \sqrt{\mathbb{R}} : [0, +\infty)$

د) $y = (x^5 - 4x^4 + 3x + 1)^2$ $R_f: [0, +\infty)$

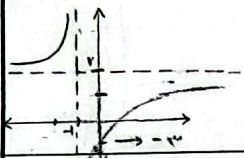
الف) $y = \frac{cx+1}{x-2} \quad \frac{ad \neq bc, c \neq 0}{-4 \neq 1, 1 \neq 0} \rightarrow R_f: \mathbb{R} - \left\{ \frac{a}{c} \right\} \rightarrow R_f: \mathbb{R} - \{2\}$

ب) $y = \frac{cx+1}{x+1} \quad \frac{ad \neq bc, c \neq 0}{1 \neq 1, 1 \neq 0} \rightarrow R_f: \mathbb{R} - \left\{ \frac{a}{c} \right\} \rightarrow R_f: \mathbb{R} - \{1\}$

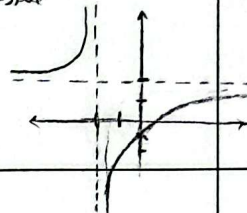
الف) $y = \sqrt{\frac{cx+1}{x+1}} \quad \frac{ad \neq bc, c \neq 0}{1 \neq 1, 1 \neq 0} \rightarrow R_f: \sqrt{\mathbb{R} - \{1\}} \rightarrow R_f: [0, +\infty) - \{\sqrt{1}\}$

ب) $y = \sqrt{\frac{cx+1}{-x+1}} \quad \frac{ad \neq bc, c \neq 0}{1 \neq -1, 1 \neq 0} \rightarrow R_f: \sqrt{\mathbb{R} - \{-1\}} \rightarrow R_f: [0, +\infty)$

الف) $y = \frac{2x-3}{x+1}$
 جانب نام (ریشه خارج) $x+1=0 \rightarrow x=-1$
 جانب باقی $\frac{c}{a}=2 \rightarrow y=2$
 $ad-bc = 2-(-2) = 4 > 0 \rightarrow \text{دری}$



ب) $y = \frac{2x-1}{x+2}$
 جانب نام (ریشه خارج) $x+2=0 \rightarrow x=-2$
 جانب باقی $\frac{c}{a}=2 \rightarrow y=2$
 $ad-bc = 4+1 = 5 > 0 \rightarrow \text{دری}$



الف) $y = \sin x + \frac{1}{\sin x}$ $-1 \leq \sin x \leq 1 \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

ب) $y = \frac{x^4+1}{x^3} = x + \frac{1}{x^3}$ $\frac{1}{x^3} < 0 \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\frac{1}{\sqrt{x}} > 0 \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\frac{1}{\sqrt{x}} > 0 \rightarrow R_f = [1, +\infty)$

الف) $y = (x^2 + \frac{1}{x^2+r} + r) - r = y = (x^2+r + \frac{1}{x^2+r}) - r \xrightarrow{x^2+r > 0} R_f = [\frac{1}{r}, +\infty) - r \rightarrow R_f = [\frac{1}{r}, +\infty)$

$y = x^2+r + \frac{1}{x^2+r} \xrightarrow{x=0} r + \frac{1}{r} = \frac{10}{r} \rightarrow R_f = [\frac{10}{r}, +\infty)$

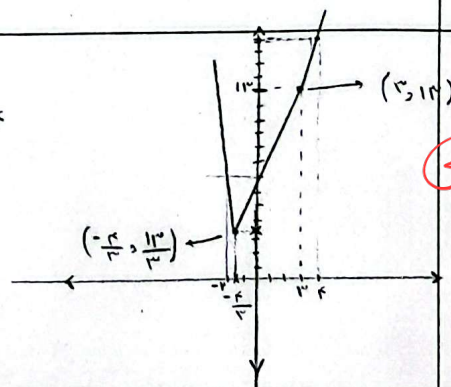
ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \frac{x^2+4+1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \xrightarrow{\sqrt{x^2+4} > 0} R_f = [\frac{5}{2}, +\infty)$

نقطه بحرانی: $x=0 \rightarrow y = \sqrt{4} + \frac{1}{\sqrt{4}} = 2 + \frac{1}{2} = \frac{5}{2}$

$y = |x-3| + |3x+4|$
 $x=3$ $x=-\frac{4}{3}$

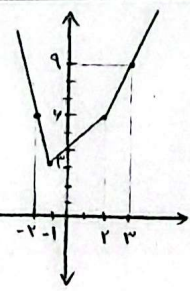
$x < -\frac{4}{3}$	$-\frac{4}{3} < x < 3$	$x > 3$
$-x+3-3x-4 = -4x-1$	$3x+3-3x-4 = -1$	$x-3+3x+4 = 4x+1$
$y = -4x-1$	$y = -1$	$y = 4x+1$

$\begin{cases} x = -2 \rightarrow y = 7 \\ x = -\frac{4}{3} \rightarrow y = \frac{13}{3} \\ x = 3 \rightarrow y = 13 \\ x = 4 \rightarrow y = 17 \end{cases}$



الف) $y = \frac{|x-2|}{x+2} + \frac{|2x+1|}{x-1}$

$\begin{cases} x = -2 \rightarrow y = 4+2=6 \rightarrow \frac{6}{-1} \\ x = -1 \rightarrow y = 3 \rightarrow \frac{3}{-2} \\ x = 2 \rightarrow y = 4 \rightarrow \frac{4}{3} \\ x = 3 \rightarrow y = 1+8=9 \rightarrow \frac{9}{2} \end{cases}$

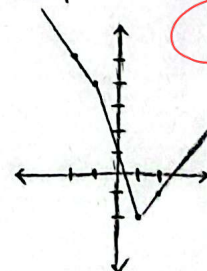


$R_f = [1, +\infty)$

ب) $y = \frac{|2x-2|}{x+1} - \frac{|x+1|}{x-1}$

$\begin{cases} x = -2 \rightarrow y = 4-1=3 \\ x = -1 \rightarrow y = 2 \\ x = 1 \rightarrow y = -2 \\ x = 2 \rightarrow y = 2-3=-1 \end{cases}$

$R_f = [-1, +\infty)$



$\frac{-1}{x+2} + \frac{2}{x+1}$

$\frac{-1}{2x+1} - \frac{1}{x-1}$

ضابطه بندی