

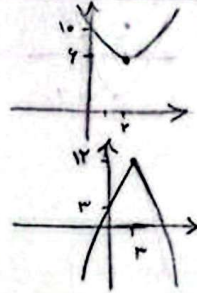
الف) $y = x^3 - 3x^2 + 3x \xrightarrow{+1-1} y_2 (x-1)^3 + 1 \rightarrow (x-1)^3 = y-1 \rightarrow x-1 = \sqrt[3]{y-1}$
 $\Rightarrow x = \sqrt[3]{y-1} + 1 \rightarrow |R_f = \mathbb{R}|$

ب) $y = \frac{1}{x^2 + 2x} \rightarrow yx^2 + 2yx = 1 \rightarrow yx^2 + 2yx - 1 = 0 \rightarrow x = \frac{-2y \pm \sqrt{4y^2 + 4y}}{2y}$
 $\Rightarrow 4y^2 + 4y \geq 0 \rightarrow 4y(y+1) \geq 0 \rightarrow \begin{matrix} -1 & 0 \\ + & - & + \end{matrix} \rightarrow (-\infty, -1] \cup [0, +\infty)$
 $y \neq 0 \rightarrow y \neq 0 \rightarrow \left. \begin{matrix} y \\ y \end{matrix} \right\} \rightarrow |R_f = (-\infty, -1] \cup [0, +\infty)|$

الف) $y = x^2 - 4x + 6$

$\min \left| \begin{matrix} \frac{-b}{2a} = \frac{-(-4)}{2} = 2 \\ (2)^2 - 4(2) + 6 = 2 \end{matrix} \right.$

$\Rightarrow$



$|R_f = [2, +\infty)|$

ب) $y = x^2 - 4x + 3$

$\max \left| \begin{matrix} \frac{-b}{2a} = \frac{-(-4)}{2} = 2 \\ -(2)^2 + 4(2) + 3 = 11 \end{matrix} \right.$

$|R_f = (-\infty, 11]|$

ج) $y = \sqrt{x^2 - 4x + 3}$

$\min \left| \begin{matrix} \frac{-b}{2a} = \frac{-(-4)}{2} = 2 \\ (2)^2 - 4(2) + 3 = -1 \end{matrix} \right.$

$\Rightarrow \sqrt{[-1, +\infty)} \Rightarrow |R_f = [0, +\infty)|$

د) $y = \sqrt{4x - x^2}$

$\max \left| \begin{matrix} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ 4(2) - (2)^2 = 4 \end{matrix} \right.$

$\Rightarrow \sqrt{(-\infty, 4]} \Rightarrow |R_f = [0, 2]|$

الف) $y = x^2 - 4x + 2x + 1$

$\Rightarrow |R_f = \mathbb{R}|$

ب) $y = x^2 - 4x^2 + 2x + 1 \Rightarrow |R_f = \mathbb{R}|$

ج) $y = \sqrt{x^2 - 4x^2 + 2x + 1}$

$\Rightarrow |R_f = [0, +\infty)|$

د) $y = (x^2 - 4x^2 + 2x + 1)^2 \Rightarrow |R_f = [0, +\infty)|$

الف) $y = \frac{x+1}{x-1}$

$\Rightarrow |R_f = \mathbb{R} - \{1\}|$

ب) $y = \frac{x+1}{x+1} \Rightarrow |R_f = \mathbb{R} - \{-1\}|$

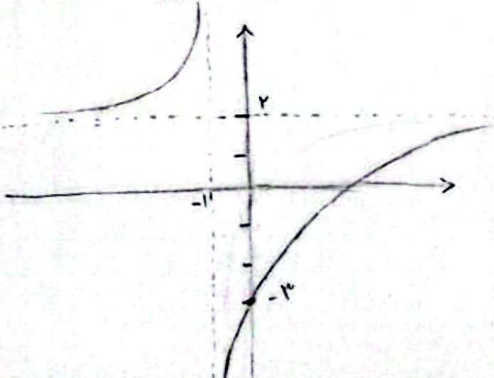
الف) $y = \sqrt{\frac{x+1}{x+1}}$

$\Rightarrow |\mathbb{R} - \{1\}| \Rightarrow |R_f = [0, +\infty) - \{\sqrt{1}\}|$

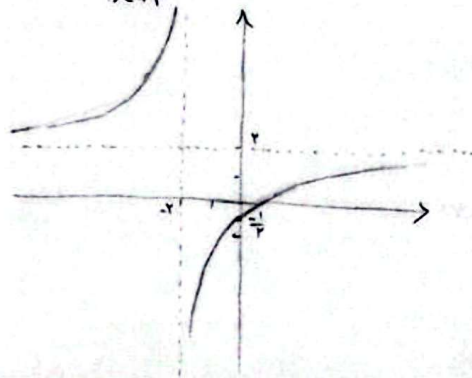
ب) $y = \sqrt{\frac{x+1}{x-1}}$

$\Rightarrow |\mathbb{R} - \{-1\}| \Rightarrow |R_f = [0, +\infty)|$

الف) $y = \frac{x-1}{x+1}$



ب) $y = \frac{x-1}{x+1}$



الف) $y = \sin x + \frac{1}{\sin x}$ $-1 \leq \sin x \leq 1$ $\Rightarrow |R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^4+1}{x^2} = x^2 + \frac{1}{x^2}$ $\rightarrow$ استيعاب $\rightarrow |R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\rightarrow |R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\rightarrow \sqrt{x} \geq 0 \rightarrow |R_f = [2, +\infty)$

الف) $y = x^r + \frac{1}{x^r+r}$ $\xrightarrow{x=r} y = \frac{r^r+r}{a} + \frac{1}{r^r+r} - r \rightarrow$ استيعاب $\rightarrow |R_f = [\frac{1}{r}, +\infty)$

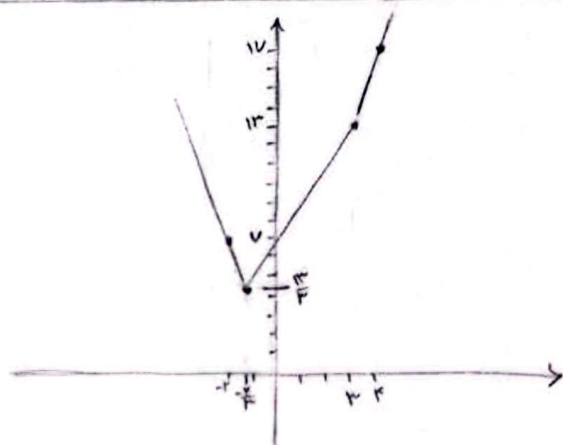
ب) $y = \frac{x^r+a}{\sqrt{x^r+r}} = \frac{x^r+r+1}{\sqrt{x^r+r}} = \frac{\sqrt{x^r+r}}{a} + \frac{1}{\sqrt{x^r+r}} \rightarrow$ استيعاب $\rightarrow |R_f = [\frac{a}{r}, +\infty)$

$y = |x-r| + |x+r|$

$x-r = x-r$ $\rightarrow -x-r$ $\rightarrow -x-r-1$ $\rightarrow -x-r-1$ $\rightarrow -x-r-1$

$x-r = x-r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$

$x-r = x-r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$

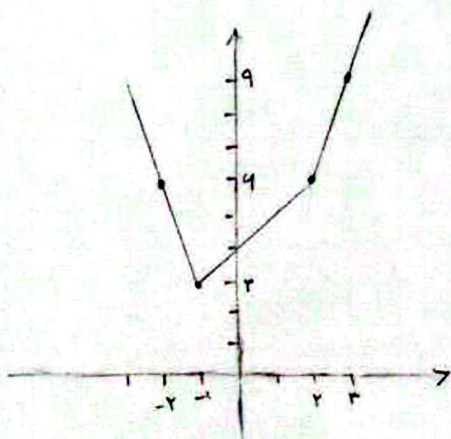


الف) $y = |x-r| + |x+r|$

$x-r = x-r$ $\rightarrow -x-r$ $\rightarrow -x-r-1$ $\rightarrow -x-r-1$ $\rightarrow -x-r-1$

$x-r = x-r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$

$x-r = x-r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$



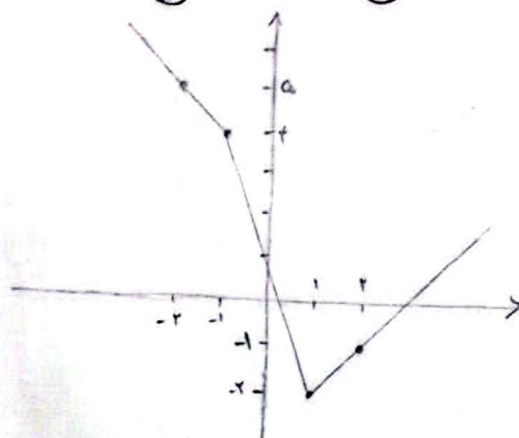
$|R_f = [r, +\infty)$

ب) $y = |x-r| - |x+r|$

$x-r = x-r$ $\rightarrow -x-r$ $\rightarrow -x-r-1$ $\rightarrow -x-r-1$ $\rightarrow -x-r-1$

$x-r = x-r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$

$x-r = x-r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$ $\rightarrow x-r+r$



$|R_f = [-r, +\infty)$