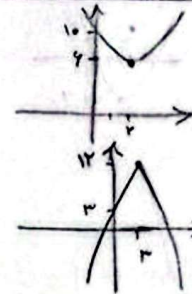


الف) $y = x^3 - 3x^2 + 3x \xrightarrow{+1-1} y_2 (x-1)^3 + 1 \rightarrow (x-1)^3 = y_2 - 1 \rightarrow x-1 = \sqrt[3]{y_2 - 1}$
 $\Rightarrow x = \sqrt[3]{y_2 - 1} + 1 \rightarrow |R_f = \mathbb{R}|$

ب) $y = \frac{1}{x^2 + 2x} \rightarrow yx^2 + 2yx = 1 \rightarrow yx^2 + 2yx - 1 = 0 \rightarrow x = \frac{-2y \pm \sqrt{4y^2 + 4y}}{2y}$
 $\Rightarrow 4y^2 + 4y \geq 0 \rightarrow 4y(y+1) \geq 0 \rightarrow \begin{matrix} -1 & 0 \\ + & - & + \end{matrix} \rightarrow (-\infty, -1] \cup [0, +\infty)$
 $y \neq 0 \rightarrow y \neq 0$
 $\Rightarrow |R_f = (-\infty, -1] \cup [0, +\infty)|$

الف) $y = x^2 - 4x + 6$

min $\left| \begin{matrix} \frac{-b}{2a} = \frac{-(-4)}{2} = 2 \\ (2)^2 - 4(2) + 6 = 2 \end{matrix} \right|$

$\Rightarrow$  $|R_f = [2, +\infty)|$

ب) $y = x^2 - 4x + 3$

max $\left| \begin{matrix} \frac{-b}{2a} = \frac{-(-4)}{2} = 2 \\ -(2)^2 + 4(2) + 3 = 11 \end{matrix} \right|$

$\Rightarrow$  $|R_f = (-\infty, 11]|$

ج) $y = \sqrt{x^2 - 4x + 3}$

min $\left| \begin{matrix} \frac{-b}{2a} = \frac{-(-4)}{2} = 2 \\ (2)^2 - 4(2) + 3 = -1 \end{matrix} \right|$

$\Rightarrow \sqrt{[-1, +\infty)} \Rightarrow |R_f = [0, +\infty)|$

د) $y = \sqrt{4x - x^2}$

max $\left| \begin{matrix} \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ 4(2) - (2)^2 = 4 \end{matrix} \right|$

$\Rightarrow \sqrt{(-\infty, 4]} \Rightarrow |R_f = [0, 2]|$

الف) $y = x^2 - 4x + 2x + 1 \rightarrow |R_f = \mathbb{R}|$

ب) $y = x^2 - 4x^2 + 2x + 1 \Rightarrow |R_f = \mathbb{R}|$

ج) $y = \sqrt{x^2 - 4x^2 + 2x + 1} \Rightarrow |R_f = [0, +\infty)|$

د) $y = (x^2 - 4x^2 + 2x + 1)^2 \Rightarrow |R_f = [0, +\infty)|$

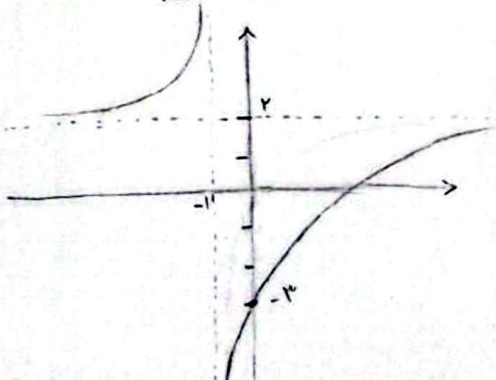
الف) $y = \frac{x+1}{x-1} \Rightarrow |R_f = \mathbb{R} - \{1\}|$

ب) $y = \frac{x+1}{x+1} \Rightarrow |R_f = \mathbb{R} - \{-1\}|$

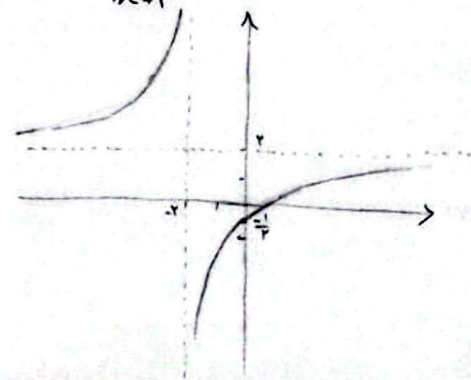
الف) $y = \sqrt{\frac{x+1}{x+1}} \rightarrow \sqrt{\mathbb{R} - \{-1\}} \Rightarrow |R_f = [0, +\infty) - \{\sqrt{-1}\}|$

ب) $y = \sqrt{\frac{x+1}{x-1}} \Rightarrow \sqrt{\mathbb{R} - \{1\}} \Rightarrow |R_f = [0, +\infty)|$

الف) $y = \frac{x-1}{x+1}$



ب) $y = \frac{x-1}{x+2}$



الف) $y = \sin x + \frac{1}{\sin x}$ $-1 \leq \sin x \leq 1 \Rightarrow |R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^4+1}{x^2} = x^2 + \frac{1}{x^2} \rightarrow \frac{-2}{x^2} \leq \frac{1}{x^2} \leq 2 \rightarrow |R_f = (-\infty, -2] \cup [2, +\infty)$

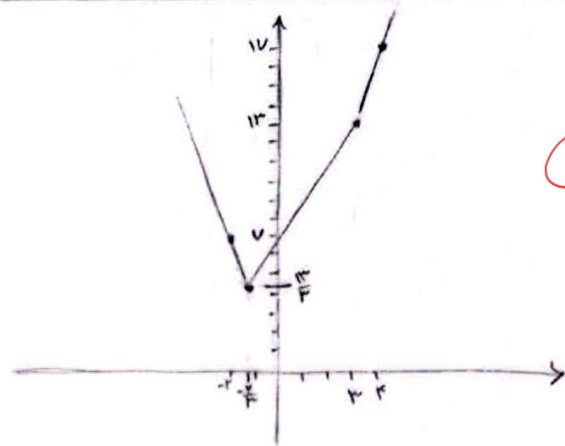
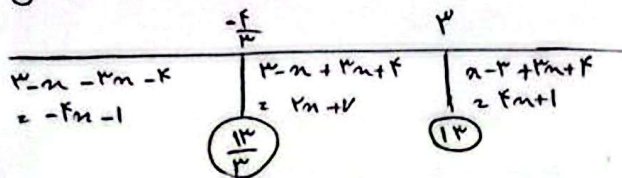
ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow |R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow \sqrt{x} \geq 0 \rightarrow |R_f = [2, +\infty)$

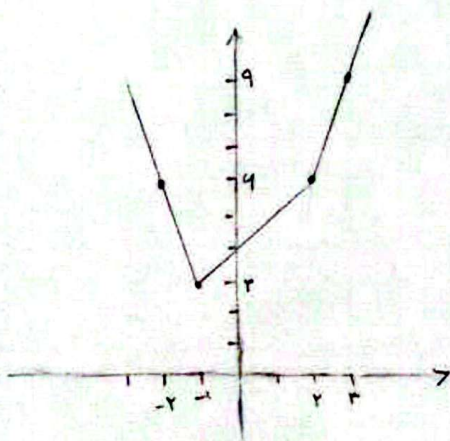
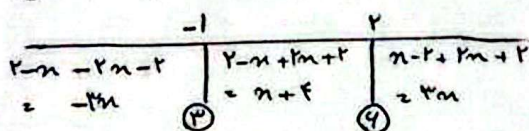
الف) $y = x^r + \frac{1}{x^r+r} \xrightarrow{x=r} y = \frac{r^r+r}{a} + \frac{1}{r^r+r} = r \rightarrow \text{اكثر من مقدار } r \rightarrow |R_f = [\frac{1}{r}, +\infty)$

ب) $y = \frac{x^r+a}{\sqrt{x^r+r}} = \frac{x^r+r+1}{\sqrt{x^r+r}} = \frac{\sqrt{x^r+r}}{a} + \frac{1}{\sqrt{x^r+r}} \rightarrow \text{اكثر من مقدار } r \Rightarrow |R_f = [\frac{a}{r}, +\infty)$

$y = |x-r| + |x+r|$

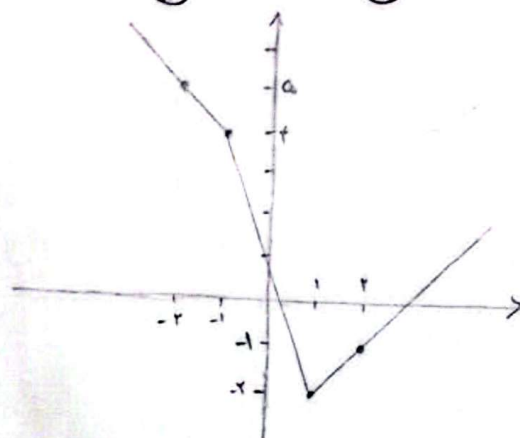
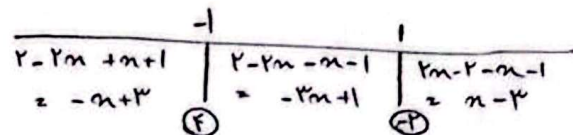


الف) $y = |x-r| + |x+r|$



$|R_f = [r, +\infty)$

ب) $y = |x-r| - |x+r|$



$|R_f = [-r, +\infty)$