۱- الف) $y = x^3 - 3x^2 + 3x \xrightarrow{-1+1} (x-1)^3 + 1 = y \Rightarrow (x-1)^3 = y-1 \Rightarrow x-1 = \sqrt[3]{y-1} \Rightarrow x = \sqrt[3]{y-1} + 1$

$R_y = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \Rightarrow 1 = yx^2 - 2yx \Rightarrow yx^2 - 2yx - 1 = 0$ $\quad \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$

$4y^2 + 4y \geq 0 \Rightarrow y^2 + y \geq 0 \Rightarrow y(y+1) \geq 0$ $\quad \frac{-1 \quad 0}{+ \; 0 \; - \; 0 \; +}$ $\rightarrow R_y = (-\infty, -1] \cup [0, +\infty)$

۲- الف) $y = x^2 - 4x + 10$ $\quad \left| \begin{array}{l} \frac{4}{2} = 2 \\ 4 - 8 + 10 = 6 \end{array} \right.$ $\quad R_y = [6, +\infty)$

ب) $y = -x^2 + 4x + 8$ $\quad \left| \begin{array}{l} \frac{-4}{-2} = 2 \\ -4 + 8 + 8 = 12 \end{array} \right.$ $\quad R_y = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x + 3}$ $\quad \left| \begin{array}{l} \frac{4}{2} = 2 \\ 4 - 8 - 1 = -5 \end{array} \right.$ $\quad \sqrt{[-5, +\infty)} = [0, +\infty)$

د) $y = \sqrt{4x - x^2}$ $\quad \left| \begin{array}{l} \frac{-4}{-2} = 2 \\ 16 - 8 = 8 \end{array} \right.$ $\quad \sqrt{(-\infty, 8]} = [0, 2]$

۳- الف) $y = x^3 - 5x^2 + 2x + 1$ $\quad R_y = \mathbb{R}$

ب) $y = x^4 - 4x^3 + 2x + 2$ $\quad R_y = \mathbb{R}$

ج) $y = \sqrt{x^4 - 4x^2 + 5x + 1}$ $\quad R_y = [0, +\infty)$

د) $y = (x^3 - 4x^2 + 2x + 1)^2$ $\quad R_y = [0, +\infty)$

۴- الف) $y = \frac{3x+1}{x-2}$ $\quad R_y = \mathbb{R} - \{3\}$

ب) $y = \frac{2x+1}{x+1}$ $\quad R_y = \mathbb{R} - \{2\}$

۵- الف) $y = \sqrt{\frac{2x+4}{x+1}}$ $\quad R_y = [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\frac{3x+1}{2-x}} = \sqrt{\frac{3x+1}{-x+2}}$ $\quad R_y = [0, +\infty) - \{\sqrt{-3}\} \Rightarrow R_y = [0, +\infty)$

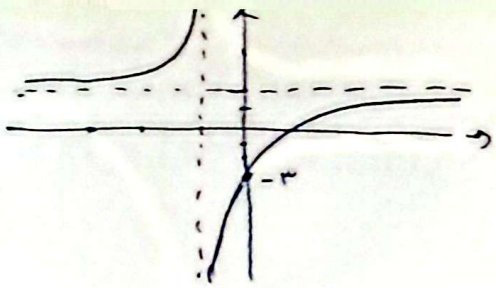
$\sqrt{-3} \notin \mathbb{R}$

-٦

الف) $y = \frac{2x-3}{x+1} \xrightarrow{x\to\infty} \frac{-3}{1} = -3$

خط مجانب افقی $= 2$

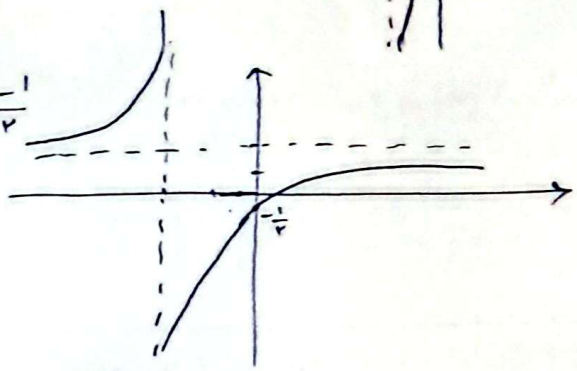
خط مجانب قائم $= -1$



ب) $y = \frac{2x-1}{x+2} \xrightarrow{x\to\infty} \frac{-1}{2}$

خط مجانب افقی $= 2$

خط مجانب قائم $= -2$



-٧

الف) $y = \sin x + \frac{1}{\sin x}$ $\quad R_y = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^6+1}{x^3} = x^3 + \frac{1}{x^3}$ $\quad R_y = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$ $\quad R_y = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\quad R_y = [2, +\infty)$

-٨

الف) $y = x^2 + \frac{1}{x^2+2} + 2 - 3 = x^2 + 2 + \frac{1}{x^2+2} - 3 \Rightarrow 2 + \frac{1}{3} - 3 = \frac{1}{3}$

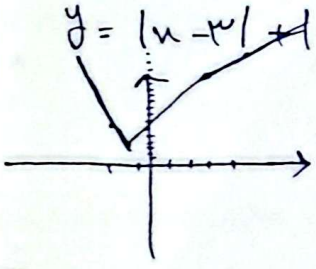
$R_y = [\frac{1}{3}, +\infty)$

ب) $\frac{x+2}{\sqrt{x^2+4}}$ ، $\frac{x^2+4+1}{\sqrt{x^2+4}} = \frac{x^2+4}{\sqrt{x^2+4}} + \frac{1}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \xrightarrow{x\to\infty} \sqrt{\infty} + \frac{1}{\sqrt{\infty}}$

$2 + \frac{1}{2} \le \frac{5}{2} \quad \Rightarrow R_y \le [\frac{5}{2}, +\infty)$

-٩

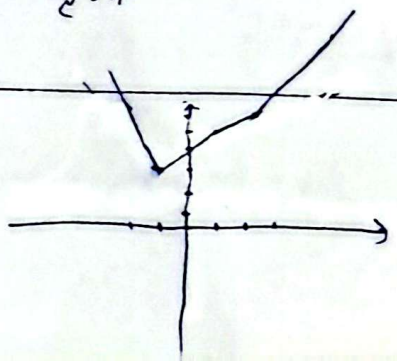
$y = |x-3| + |2x+4|$



$-\frac{4}{2}$ $\quad 3$

$-x+2-x-2$	$-x+2+x+2$	$x-2+2x+4$
$-3x-1$	$2x+7$	$4x+1$
-2	$-\frac{4}{2} = -1,2$	3
7	$\frac{14}{2} = 4,2$	11

$-4(\frac{-4}{2}) - 1 = \frac{14}{2}$



-١٠

الف) $y = |x-2| + |2x+2|$

-1 $\quad 2$

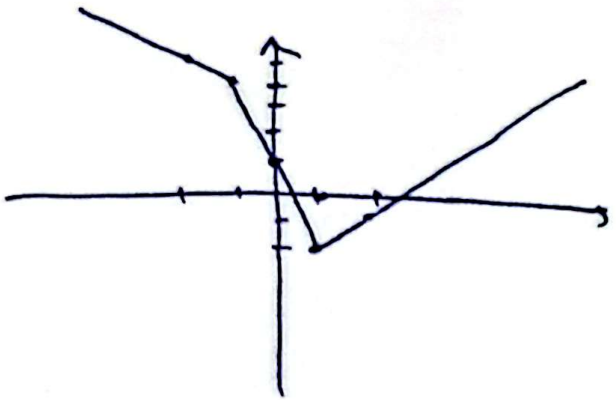
$-x+2-2x-2$	$-x+2+2x+2$	$x-2+2x+2$
$-3x$	$x+4$	$3x$
-2	-1	0
4	3	5

$R_y = [3, +\infty)$

ب) $y = |2x-2| - |x+1|$

$x < -1$	$-1 \le x < 1$	$x \ge 1$
$-2x+2+x+1$	$-2x+2-x-1$	$2x-2-x-1$
$-x+3$	$-3x+1$	$x-3$

x	-2	-1	0	1	2
y	5	4	1	-2	-1



$R_y = [-2, +\infty)$