ریاضی ۱۹,۲۵

۱) (۵)

الف) $y = x^3 - 3x^2 + 3x + 1 - 1 \longrightarrow y = (x-1)^3 + 1$

$y - 1 = (x-1)^3 \longrightarrow \sqrt[3]{y-1} = x - 1$

$x = \sqrt[3]{y-1} + 1 \longrightarrow R_f = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 2x} \qquad yx^2 - 2yx - 1 = 0 \longrightarrow \Delta = 4y^2 - 4(-1)(y) = 4y^2 + 4y$

$x = \frac{+2y \pm \sqrt{4y^2 + 4y}}{2y} \qquad 4y^2 + 4y \geq 0$

$4y(y+1) \geq 0 \qquad \frac{-1 \quad 0}{+ \; - \; +}$

$\longrightarrow y \neq 0$ (II)

(I), (II) $\longrightarrow R_f = (-\infty, -1] \cup (0, +\infty)$ $\qquad (-\infty, -1] \cup [0, +\infty)$ (I)

۲) (۵)

الف) $y = x^2 - 4x + 10 \qquad x_{min} = \frac{-b}{2a} = 2 \;,\; y = 2 \qquad R_f = [2, +\infty)$

ب) $y = -x^2 + 2x + 2 \qquad x_{max} = \frac{-b}{2a} = 1 \;,\; y = 12 \qquad R_f = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 5} \qquad x_{min} = \frac{-b}{2a} = 2 \;,\; y = -7 \qquad R_f = \sqrt{[-7, +\infty)} \longrightarrow [0, +\infty)$

د) $y = \sqrt{2x - x^2} \qquad x_{max} = \frac{-b}{2a} = 1 \;,\; y = 9 \qquad R_f = \sqrt{(-\infty, 9]} \longrightarrow [0, 3]$

$-x^2 + 2x$

۳) (۵)

الف) $y = x^3 - 3x^2 + 2x + 1 \longrightarrow R_f = \mathbb{R}$

ب) $y = x^5 - 2x^4 - 4x + 2 \longrightarrow R_f = \mathbb{R}$

ج) $y = \sqrt{x^7 - 2x^4 + 3x + 1} \longrightarrow R_f = [0, +\infty)$

د) $y = (x^{10} - 4x^2 + 2x + 1)^3 \longrightarrow R_f = [0, +\infty)$

۴) (۵)

الف) $y = \frac{2x+1}{x-2} \qquad R_f = \mathbb{R} - \{2\}$

ب) $y = \frac{2x+1}{x+1} \qquad R_f = \mathbb{R} - \{2\}$

③

الف) $y=\sqrt{\frac{2x+4}{x-1}}$ $\quad R_f=\sqrt{\mathbb{R}-\{2\}} \rightarrow [0,+\infty)-\{2\}$

ب) $y=\sqrt{\frac{3x+1}{2-x}}$ $\quad R_f=\sqrt{\mathbb{R}-\{-3\}} \rightarrow [0,+\infty)$

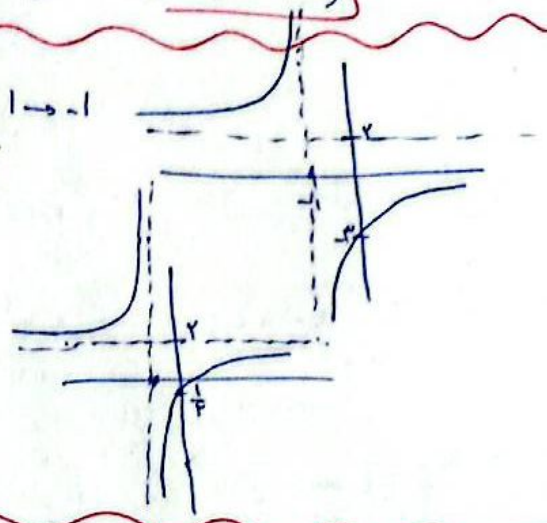
④

الف) $y=\frac{2x-3}{x+1}$ $\quad$ مجانب قائم $=x+1\rightarrow -1$ $\quad$ مجانب افقی $=2$

$x=0 \rightarrow x=-3$

ب) $y=\frac{2x-1}{x+2}$ $\quad$ مجانب قائم $=-2$ $\quad$ مجانب افقی $=2$

$\hookrightarrow x=0 \rightarrow -\frac{1}{2}$



⑦

الف) $y=\sin x+\frac{1}{\sin x}$ $\quad R_f=(-\infty,-2]\cup[2,+\infty)$

ب) $y=\frac{x^2+1}{x^2} \rightarrow x^2+\frac{1}{x^2} \rightarrow R_f=(-\infty,-2]\cup[2,+\infty)$

ج) $y=\frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} \Rightarrow \sqrt[3]{\frac{x^2}{x}}+\frac{1}{\sqrt[3]{x}} \rightarrow \sqrt[3]{x}+\frac{1}{\sqrt[3]{x}} \rightarrow R_f=(-\infty,-2]\cup[2,+\infty)$

د) $y=\sqrt{x}+\frac{1}{\sqrt{x}} \rightarrow R_f=[2,+\infty)$

⑧

الف) $y=x^2+\frac{1}{x^2+3} \rightarrow x^2+3+\frac{1}{x^2+3}-3$ $\quad \hookrightarrow x=0 \rightarrow R=[\frac{1}{3},+\infty)$

$t+\frac{1}{t}-3 \rightarrow t+\frac{1}{t}-3\geq 2 \rightarrow t+\frac{1}{t}\geq 5$

$t+\frac{1}{t}\geq 2$

$\hookrightarrow R_f=[5,+\infty)$

ب) $y=\frac{x^2+5}{\sqrt{x^2+4}} \rightarrow \frac{x^2+4+1}{\sqrt{x^2+4}} \rightarrow \sqrt{x^2+4}+\frac{1}{\sqrt{x^2+4}} \rightarrow 2+\frac{1}{2}=\frac{5}{2} \rightarrow R_f=[\frac{5}{2},+\infty)$

9

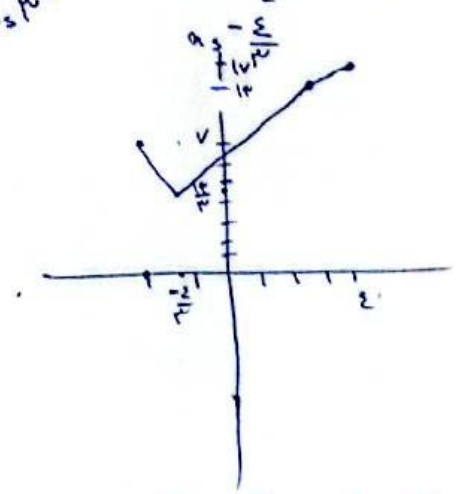
$y = |x-2| + |3x+6|$

$x = 2$, $x = -2$

$x > 2$: $x - 2 + 3x + 6 = 4x + 4$

$-2 \le x \le 2$: $-x + 2 + 3x + 6 = 2x + 8$

$x < -2$: $-x + 2 - 3x - 6 = -4x - 4$



5

$x = \frac{-3}{2} \to 2 \times \frac{-3}{2} + 8 = 5$

$x = 2 \to 2 \times 2 + 8 = 12$

$x = -2 \to -4 \times -2 - 4 = 4$

10

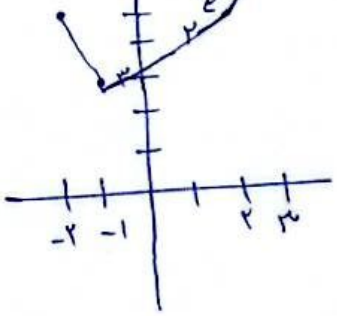
الف) $y = |x-2| + |2x+2|$

$x = 2$, $x = -1$

$x > 2$: $x - 2 + 2x + 2 = 3x$

$-1 \le x \le 2$: $-x + 2 + 2x + 2 = x + 4$

$x < -1$: $-x + 2 - 2x - 2 = -3x$



$R_f = [3, +\infty)$

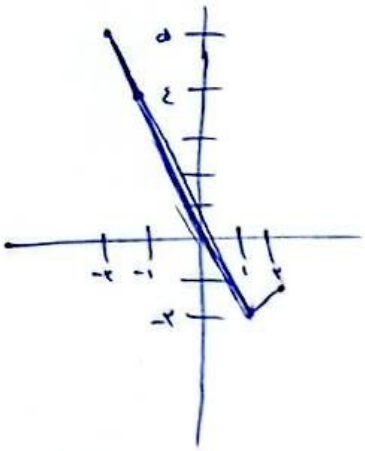
ب) $y = |2x-2| - |x+1|$

$x = 1$, $x = -1$

$x > 1$: $2x - 2 - x - 1 = x - 3$

$-1 \le x \le 1$: $-2x + 2 - x - 1 = -3x + 1$

$x < -1$: $-2x + 2 + x + 1 = -x + 3$



$R_f = [-3, +\infty)$