

الف) $y = x^3 - 3x^2 + 3x \rightarrow y = x^3 - 3x^2 + 3x - 1 + 1 \rightarrow y = (x-1)^3 + 1 \rightarrow (x-1)^3 = y-1 \rightarrow x-1 = \sqrt[3]{y-1} \rightarrow x = \sqrt[3]{y-1} + 1$
 $R_y = \mathbb{R}$

ب) $y = \frac{1}{x^2 - 3x} \rightarrow y = \frac{1}{x^2 - 3x + 1 - 1} \rightarrow y = \frac{1}{(x-1)^2 - 1} \rightarrow y(x-1)^2 - y = 1 \rightarrow y(x-1)^2 = 1+y \rightarrow (x-1)^2 = \frac{1+y}{y}$
 $x-1 = \pm \sqrt{\frac{1+y}{y}} \rightarrow x = \pm \sqrt{\frac{1+y}{y}} + 1$

$\frac{1+y}{y} \geq 0 \rightarrow \frac{-1}{1-\frac{1}{y}} \geq 0 \rightarrow R_y = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 1 \rightarrow x_{min} = \frac{b}{2a} = \frac{4}{2} = 2 \rightarrow y_{min} = 4 - 16 + 1 = -11 \rightarrow R_y = [-11, +\infty)$

ب) $y = -x^2 + 4x + 3 \rightarrow x_{max} = \frac{4}{-2} = -2 \rightarrow y_{min} = -4 + 16 + 3 = 15 \rightarrow R_y = (-\infty, 15]$

ج) $y = \sqrt{x^2 - 4x + 3} \rightarrow x_{min} = \frac{4}{2} = 2 \rightarrow y_{min} = \sqrt{4 - 16 + 3} = \sqrt{-9} \rightarrow R_y = \sqrt{-9, +\infty} = [0, +\infty)$

د) $y = \sqrt{4x - x^2} \rightarrow x_{max} = \frac{4}{2} = 2 \rightarrow y_{max} = \sqrt{16 - 4} = \sqrt{12} \rightarrow R_y = \sqrt{0, 12} = [0, 2\sqrt{3}]$

الف) $y = x^3 - 2x^2 + 3x + 1 \rightarrow R_y = \mathbb{R}$

ب) $y = x^3 - 4x^2 + 3x + 2 \rightarrow R_y = \mathbb{R}$

ج) $y = \sqrt{x^3 - 4x^2 + 3x + 1} \rightarrow R_y = \sqrt{(-\infty, +\infty)} = [0, +\infty)$

د) $y = (x^3 - 4x^2 + 3x + 1)^2$

$y \geq 0 \rightarrow R_y = [0, +\infty)$

الف) $y = \frac{x+1}{x-1} \rightarrow R_y = \mathbb{R} - \left\{ \frac{1}{2} \right\}$

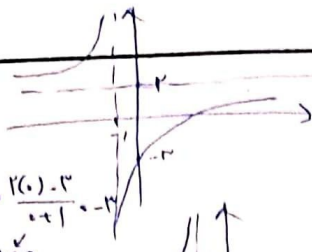
ب) $y = \frac{x+1}{x+1} \rightarrow R_y = \mathbb{R} - \{1\}$

الف) $y = \sqrt{\frac{x+1}{x+1}} \rightarrow R_y = \sqrt{\mathbb{R} - \{1\}} \rightarrow R_y = [0, +\infty) - \{1\}$

ب) $y = \sqrt{\frac{x+1}{-x+1}} \rightarrow R_y = \sqrt{(-\infty, +\infty) - \{1\}} = [0, +\infty)$

الف) $y = \frac{x^2-2}{x+1}$

مخرج $x+1$ جذر x^2-2



ب) $y = \frac{x-1}{x+1}$

مخرج $x+1$ جذر $x-1$



الف) $y = \sin x + \frac{1}{\sin x}$ $R_y: (-\infty, -1] \cup [1, +\infty)$

ب) $y = \frac{x^2+1}{x^2} = 1 + \frac{1}{x^2} \rightarrow R_y: (-\infty, -1] \cup [1, +\infty)$

ج) $y = \frac{\sqrt{x^2+1}}{\sqrt{x}} = \sqrt{x + \frac{1}{x}} \rightarrow R_y: (-\infty, -1] \cup [1, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}} \rightarrow R_y: [1, +\infty)$

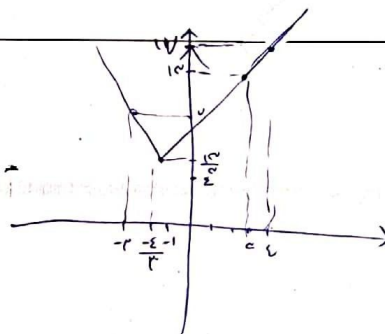
الف) $y = x^2 + \frac{1}{x^2+2} \rightarrow y = \frac{x^4+x^2+1}{x^2+2} \rightarrow R_y: [\frac{1}{2}, +\infty)$

مخرج x^2+2 جذر x^4+x^2+1

ب) $y = \frac{x^2+a}{\sqrt{x^2+2}} \rightarrow y = \frac{x^2+2}{\sqrt{x^2+2}} + \frac{a-2}{\sqrt{x^2+2}} \rightarrow R_y: [1+\frac{a-2}{\sqrt{2}}, +\infty)$

$y = |x-2| + |x+1|$

مخرج $x-2$ جذر $x+1$



الف) $y = |x-2| + |x+1|$

مخرج $x-2$ جذر $x+1$

ب) $y = |x-2| - |x+1|$

مخرج $x-2$ جذر $x+1$

