

الفصل الرابع (مقدمة)

المسألة الأولى

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(المسألة الأولى)

$$y = x^3 - 3x^2 + 4x + 1 - 1 \Rightarrow y = (x-1)^3 + 1$$

$$y-1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1$$

$$x = \sqrt[3]{y-1} + 1 \rightarrow R(f) = \mathbb{R}$$

$$\therefore y = \frac{1}{x^3 - 12} \quad yx^3 - 12x = 1 \quad \Delta \rightarrow \epsilon y^3 - \frac{\epsilon(-12y)}{\epsilon x y} = \epsilon y^3 + \epsilon y$$

$$x = \frac{-12y \pm \sqrt{144y^2 + 4\epsilon y}}{2\epsilon y} \rightarrow y \neq 0 \quad (II)$$

$$\epsilon y^3 + \epsilon y \geq 0$$

$$\epsilon y(y+1) \geq 0 \quad \begin{matrix} -1 & 0 \\ + & - & + \\ (-\infty, -1] & (-1, 0) & (0, +\infty) \end{matrix} \quad (I)$$

$$(I) \cap (II) \rightarrow R(f) = (-\infty, -1] \cup (0, +\infty)$$

المسألة الثانية

$$\text{اذا } yx^3 - 12x + 1 = 0 \Rightarrow \quad \frac{-b}{2a} = 2 \quad y = 6 - 12 + 1 = -5$$

$$R(f) = [4, +\infty)$$

$$\therefore y = -x^3 + 12x + 1 \quad \frac{-b}{2a} = 2 \quad y = -9 + 12 + 1 = 4 \rightarrow R(f) = (-\infty, 4]$$

$$b) y = \sqrt{x^3 - 12x - 3} \quad \frac{-b}{2a} = 2 \quad y_{\min} \rightarrow 8 - 12 - 3 = -7$$

$$R(f) = [-7, +\infty) \leftarrow \sqrt{(-7 + \infty)} \rightarrow [0, +\infty)$$

$$1) \sqrt{4x - x^2} \quad \frac{-b}{2a} = 2 \quad y = -4 + 12 = 8$$

$$R(f) = \sqrt{(-\infty, 4]} \rightarrow [0, 2]$$

$$\text{اذا } y = x^3 - 8x^2 + 12x + 1 \rightarrow R(f) = \mathbb{R}$$

المسألة الثالثة

$$\therefore y = x^3 - 8x^2 + 12x + 1 \rightarrow R(f) = \mathbb{R}$$

$$\therefore y = \sqrt{x^3 - 8x^2 + 12x + 1} \rightarrow R(f) = [0, +\infty)$$

$$\therefore y = (x^3 - 8x^2 + 12x + 1)^2 \quad R(f) = [0, +\infty)$$

(سوال ٤)

الف) $y = \frac{3x+1}{x-2} \rightarrow R(f) = \mathbb{R} - \{3\}$

ب) $y = \frac{(1)x+1}{(1)x+1} \rightarrow R(f) = \mathbb{R} - \{1\}$

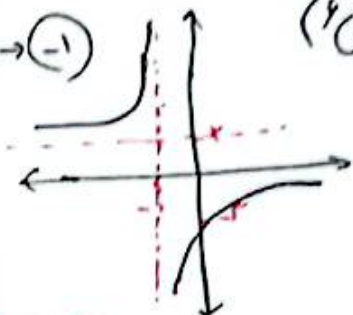
(سوال ٥)

الف) $y = \sqrt{\frac{3x+1}{x-1}} \quad R(f) = \sqrt{\mathbb{R} - \{3\}} \Rightarrow [0, +\infty) - \{2\}$

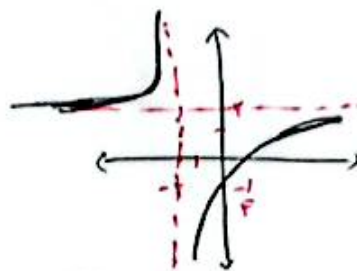
ب) $y = \sqrt{\frac{3x+1}{2-x}} \quad R(f) = \sqrt{\mathbb{R} - \{3\}} \rightarrow [0, +\infty)$

(سوال ٦)

الف) $y = \frac{2x-3}{x+1} \rightarrow x \rightarrow -1 \rightarrow \text{مخالف سهم}$
 $\rightarrow x \rightarrow -3 \rightarrow \text{مخالف سهم}$



ب) $y = \frac{2x-1}{x+2} \rightarrow x \rightarrow -2 \rightarrow \text{مخالف سهم}$
 $\rightarrow x \rightarrow -1 \rightarrow \text{مخالف سهم}$



(سوال ٧)

الف) $y = \sin x \cdot \frac{1}{\sin x} \quad R(f) = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^2+1}{x^3} \rightarrow x^2 + \frac{1}{x^3} \rightarrow R(f) = (-\infty, -2] \cup [2, +\infty)$

ج) $\frac{\sqrt[3]{x^2+1}}{\sqrt[3]{x}} = \sqrt[3]{\frac{x^2+1}{x}} = \sqrt[3]{\frac{x^2}{x} + \frac{1}{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \Rightarrow R(f) = (-\infty, 2] \cup [2, +\infty)$

د) $\sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow [2, +\infty) = R(f) \rightarrow R(f) = [2, +\infty)$

$$a) y = x^2 + \frac{1}{x^2+3} \rightarrow x^2 + 3 + \frac{1}{x^2+3} - 3$$

$$\frac{x^2+3}{x^2+3} + \frac{1}{x^2+3} - 3 \rightarrow x^2 + \frac{1}{x^2} - 3 \geq x^2 \rightarrow x^2 + \frac{1}{x^2} \geq 3$$

$$R(f) = [3, +\infty)$$

$$b) y = \frac{x^2+2}{\sqrt{x^2+4}} = \frac{x^2+4}{\sqrt{x^2+4}} \rightarrow \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \rightarrow x + \frac{1}{x} \geq \frac{2}{x}$$

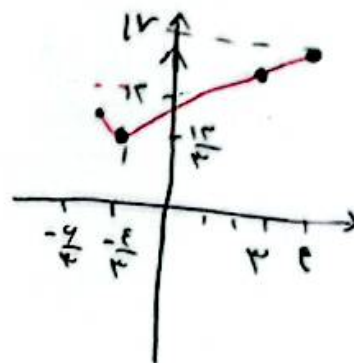
$$R(f) = \left[\frac{2}{x}, +\infty\right)$$

$$y = |x-3| + |3x+5| - \frac{5}{x}$$

$$x-3+3x+5 \rightarrow 4x+2 \quad x > 3$$

$$-x+3+3x+5 \rightarrow 2x+8 \quad -\frac{5}{x} \leq x \leq 3$$

$$-x+3-3x-5 \rightarrow -4x-2 \quad x < -\frac{5}{x}$$



سوال (1)

$$c) y = |x-2| + |2x+3| \quad -x-$$

$$x=2 \quad x=-1$$

$$x-2+2x+3 \rightarrow 3x+1 \quad x > 2$$

$$-x+2+2x+3 \rightarrow x+5 \quad -1 \leq x \leq 2$$

$$-x-2-2x-3 \rightarrow -3x-5 \quad x < -1$$



$$R(f) = [-1, +\infty)$$

سوال (1)

$$d) y = |x-2| - |x+1|$$

$$x-2$$

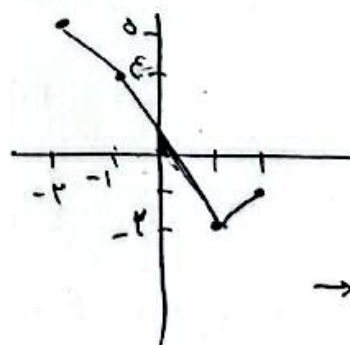
$$(1)$$

$$(-1)$$

$$x-2-x-1 \rightarrow -3 \quad x > 1$$

$$-x+2-x-1 \rightarrow -2x+1 \quad -1 \leq x \leq 1$$

$$-x+2-x+1 \rightarrow -2x+3 \quad x < -1$$



$$\rightarrow R(f) = [-1, +\infty)$$