

الفصل الرابع (مقدمة)

مقدمة

19, 20

(1, 2)

$$y = x^3 - 3x^2 + 4x + 1 - 1 \Rightarrow y = (x-1)^3 + 1$$

$$y-1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} = x-1$$

$$x = \sqrt[3]{y-1} + 1 \rightarrow R(f) = \mathbb{R}$$

$$\Rightarrow y = \frac{1}{x^3 - 3x^2 - 1} \quad yx^3 - 3yx^2 - 1 = 0 \quad \Delta \rightarrow 9y^2 - 4(-12y) - 4y = 0$$

$$x = \frac{3y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow y \neq 0 \quad (II)$$

$$4y^2 + 4y \geq 0$$

$$4y(y+1) \geq 0 \quad \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$$

$$(-1, -1] \cup [0, +\infty) \quad (I)$$

$$(I) \cap (II) \rightarrow R(f) = (-\infty, -1] \cup (0, +\infty)$$

مسألة 2

$$\text{ا) } y = x^3 - 5x + 1 \Rightarrow \frac{-b}{2a} = 2 \quad y = 8 - 10 + 1 = -1$$

$$R(f) = [4, +\infty)$$

$$\text{ب) } y = -x^3 + 4x^2 - 3 \Rightarrow \frac{-b}{2a} = 2 \quad y = -9 + 16 - 3 = -1$$

$$R(f) = (-\infty, 12]$$

$$\text{ج) } y = \sqrt{x^3 - 5x - 3} \quad \frac{-b}{2a} = 2 \quad y_{\min} \rightarrow 8 - 10 - 3 = -5$$

$$R(f) = [0, +\infty) \quad \leftarrow \sqrt{(-5 + \infty)} \rightarrow [0, +\infty)$$

$$\text{د) } \sqrt{4x - x^2} \quad \frac{-b}{2a} = 2 \quad y = -4 + 16 = 12$$

$$R(f) = \sqrt{(-\infty, 4]} \rightarrow [0, 2]$$

$$\text{هـ) } y = x^3 - 8x^2 + 2x + 1 \rightarrow R(f) = \mathbb{R}$$

مسألة 3

$$\text{ب) } y = x^3 + 4x^2 + 4x + 2 \rightarrow R(f) = \mathbb{R}$$

$$\text{ج) } \sqrt{x^3 - 4x^2 + 2x + 1} \rightarrow R(f) = [0, +\infty)$$

$$\text{د) } y = (x^3 - 4x^2 + 2x + 1)^2 \rightarrow R(f) = [0, +\infty)$$

الف) $y = \frac{3x+1}{x-2} \rightarrow R(f) = \mathbb{R} - \{3\}$

سوال ٤) ٥

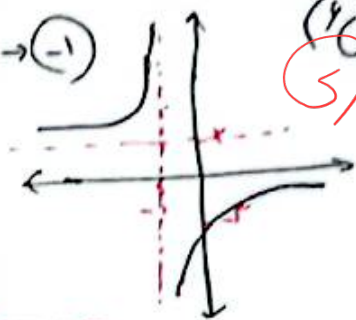
ب) $y = \frac{(1)x+1}{(1)x+1} \rightarrow R(f) = \mathbb{R} - \{1\}$

سوال ٥) ١, ٧٥

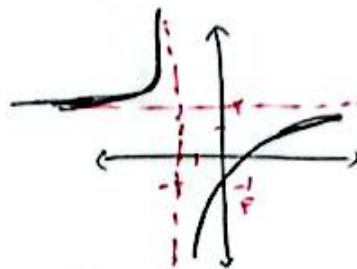
الف) $y = \sqrt{\frac{3x+1}{x-1}} \quad R(f) = \sqrt{\mathbb{R} - \{1\}} \Rightarrow [0, +\infty) - \{2\}$

ب) $y = \sqrt{\frac{3x+1}{2-x}} \quad R(f) = \sqrt{\mathbb{R} - \{2\}} \rightarrow [0, +\infty)$

الف) $y = \frac{2x-3}{x+1} \rightarrow x \rightarrow -1 \rightarrow \text{مخالف قسمة}$
 $\rightarrow x \rightarrow -3 \rightarrow \text{مخالف اتي}$



ب) $y = \frac{2x-1}{x+2} \rightarrow x \rightarrow -2 \rightarrow \text{مخالف قسمة}$
 $\rightarrow x \rightarrow 2 \rightarrow \text{مخالف اتي}$
 $x \rightarrow 2 \rightarrow \frac{1}{2}$



الف) $y = \sin x \rightarrow \frac{1}{\sin x} \quad R(f) = (-\infty, -2] \cup [2, +\infty)$

سوال ٧) ٥

ب) $y = \frac{x^2+1}{x^3} \rightarrow x^2 + \frac{1}{x^3} \rightarrow R(f) = (-\infty, -2] \cup [2, +\infty)$

ج) $\frac{\sqrt[3]{x^2+1}}{\sqrt[3]{x}} = \sqrt[3]{\frac{x^2+1}{x}} = \sqrt[3]{\frac{x^2}{x} + \frac{1}{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}} \Rightarrow R(f) = (-\infty, 5] \cup [2, +\infty)$

د) $\sqrt{x} + \frac{1}{\sqrt{x}} \Rightarrow [2, +\infty) = R(f) \rightarrow R(f) = [2, +\infty)$

1) $y = x^2 + \frac{1}{x^2 + 3} \rightarrow x^2 + 3 + \frac{1}{x^2 + 3} - 3 \rightarrow x^2 + 3 + \frac{1}{x^2 + 3} - 3 \rightarrow x^2 + \frac{1}{x^2} - 3 \geq 2 \rightarrow x^2 + \frac{1}{x^2} \geq 5$

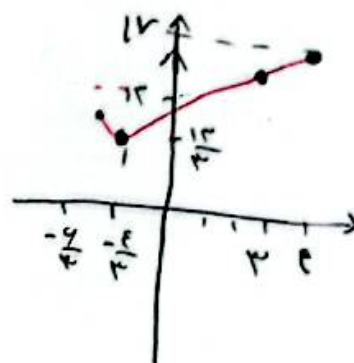
$R(x) = [2, +\infty)$

$\rightarrow y = \frac{x^2 + 2}{\sqrt{x^2 + 4}} = \frac{x^2 + 4 - 2}{\sqrt{x^2 + 4}} \rightarrow \sqrt{x^2 + 4} + \frac{1}{\sqrt{x^2 + 4}} \rightarrow x^2 + \frac{1}{x^2} \geq \frac{5}{2}$

$R(x) = [\frac{\sqrt{5}}{2}, +\infty)$

$y = |x - 3| + |3x + 4| - \frac{5}{x}$

$x - 3 + 3x + 4 \rightarrow 4x + 1 \quad x > 3$
 $-x + 3 + 3x + 4 \rightarrow 2x + 7 \quad -\frac{5}{x} \leq x \leq 3$
 $-x + 3 - 3x - 4 \rightarrow -4x - 1 \quad x < -\frac{5}{x}$

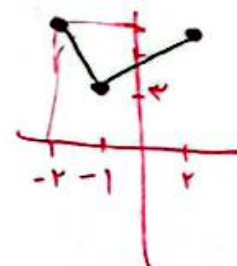


سوال (1)
5

الف) $y = |x - 2| + |2x + 3|$

$x = 2 \quad x = -1$

$3x \leftarrow x - 2 + 2x + 3 \quad x > 2$
 $x + 5 \leftarrow -x + 2 + 2x + 3 \quad -1 \leq x \leq 2$
 $-3x \leftarrow -x + 2 - 2x - 3 \quad x < -1$

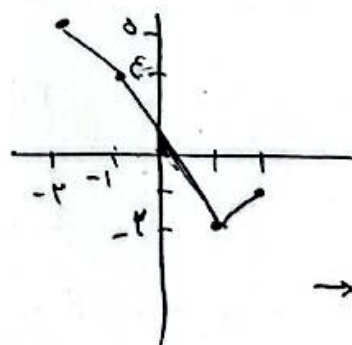


$R(x) = [-1, +\infty)$

$\rightarrow y = |4x - 2| - |x + 1|$

$x = 2 \quad x = -1$

$3x - 2 - x - 1 \quad x > 2$
 $-3x + 1 - x - 1 \quad -1 \leq x \leq 2$
 $-x + 3 - x - 1 \quad x < -1$



$\rightarrow R(x) = [-1, +\infty)$