حلول تمارين

1- أ) $x^2 - 2x + 1 = y \longrightarrow x^2 - 2x + 1 = y - 1$

$(x-1)^2 = y - 1$

$x = \sqrt{y-1} + 1$

$R_f = \mathbb{R}$.

ب) $y = \frac{1}{x^2 - 2x} \longrightarrow \frac{1}{y} = x^2 - 2x \longrightarrow \frac{1}{y} + 1 = x^2 - 2x + 1$

$\frac{1+y}{y} = (x-1)^2$

$x = \sqrt{\frac{1+y}{y}} + 1$

$\frac{1+y}{y} \geq 0$ $\quad -1 \quad 0$ $\quad + \quad - \quad +$ $\longrightarrow R_f = \mathbb{R} - (-1, 0]$

2- أ) $\min$ ext $\left| \begin{matrix} -\frac{b}{2a} = 4 \\ y \end{matrix} \right.$ $\quad R_f = [4, +\infty)$

ب) $\min$ ext $\left| \begin{matrix} -\frac{b}{2a} = 2 \\ 12 \end{matrix} \right.$ $\quad R_f = [-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 5}$ ext max $\left| \begin{matrix} -\frac{b}{2a} = 2 \\ -7 \end{matrix} \right.$ $[-7, +\infty)$

$R_f = [0, +\infty)$

د) $y = \sqrt{8x - x^2}$ ext max $\left| \begin{matrix} -\frac{b}{2a} = 4 \\ 9 \end{matrix} \right.$ $[9, +\infty)$

$R_f = [0, +\infty)$

3- أ) $R_f = \mathbb{R}$ ب) $R_f = \mathbb{R}$ ج) $y = \sqrt{\mathbb{R}} = [0, +\infty)$

د) $R_f = \mathbb{R}$

4- أ) $R_f = \mathbb{R} - \{\frac{1}{3}\}$ ب) $\mathbb{R} - \{2\}$

حل تمارين

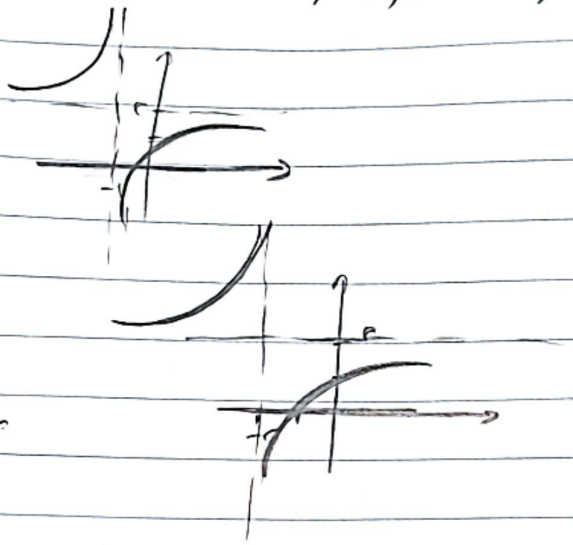
ب- $y = \sqrt{\frac{3x+5}{x+1}}$ ⟵ $y^2 = \frac{3x+5}{x+1}$ ⟵ $x = \frac{3-y^2}{y^2-1}$

$R_f = \mathbb{R} - \{1\}$ ⟵ $R_y = [0, +\infty) - \{\sqrt{3}\}$

$\frac{y^2-3}{y^2-1} \geq 0$ ⟵ $R_f = [-\infty, -1) \cup [\sqrt{3}, +\infty)$

ج- $y = \sqrt{\frac{1+3x}{2-x}}$ ⟵ $R_f = \mathbb{R} - \{-2\}$ ⟵ $R_y = [0, +\infty)$

د- $y = \frac{3x-1}{x+2}$



هـ- $y = \frac{x-1}{x+2}$

و- $R_y = [2, +\infty)$

ز- $y = x^2 + \frac{1}{x^2}$ ⟵ $R_y = (-\infty, -2] \cup [2, +\infty)$

ح- $y = x^3 + \frac{1}{x^3}$ ⟵ $R_y = (-\infty, -2] \cup [2, +\infty)$

ط- $y = \frac{1}{\sqrt{x}}$ ⟵ $R_y = [0, +\infty)$

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۸- الف) $y = (x^2+3) + \frac{1}{(x^2+3)} = 3 + \frac{1}{3} = \frac{10}{3}$ ، $t = x^2+3$

$x = 0 \rightarrow \min_t = 3$ $\quad R_y = [\frac{10}{3}, +\infty)$

$t > 0$

ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} = 2 + \frac{1}{2} = \frac{5}{2}$ ، $t = \sqrt{x^2+4}$

$x = 0 \rightarrow \min_t = \sqrt{4} = 2$ $\quad R_f = [\frac{5}{2}, +\infty)$

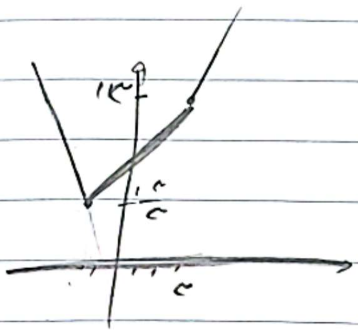
$t > 0$

۹- $y = |x-3| + |2x+4|$

$-\frac{4}{2} \qquad 3$

$3-x-2x-4 = -3x-1 \quad | \quad 3-x+2x+4 = x+7 \quad | \quad x-3+2x+4 = 3x+1$

$\left| \begin{matrix} -\frac{4}{2} \\ \frac{10}{2} \end{matrix} \right. \qquad \left| \begin{matrix} 3 \\ 10 \end{matrix} \right.$



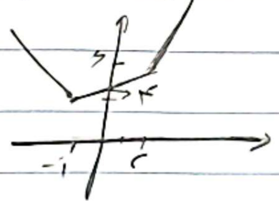
۱۰-

الف) $y = |x-2| + |2x+2|$

$-1 \qquad 2$

$-3x \quad | \quad x+4 \quad | \quad 3x$

$\left| \begin{matrix} -1 \\ 3 \end{matrix} \right. \qquad \left| \begin{matrix} 2 \\ 6 \end{matrix} \right.$



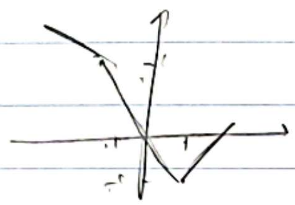
$R_f = [3, +\infty)$

ج) $y = |2x-2| - |x+1|$

$-1 \qquad 1$

$3-x \quad | \quad -3x+1 \quad | \quad x-3$

$\left| \begin{matrix} -1 \\ 4 \end{matrix} \right. \qquad \left| \begin{matrix} 1 \\ -2 \end{matrix} \right.$



$R_y = [-2, +\infty)$