یازدهم ریاضی دختر C ★ سها مهدی ★

۱)

الف) $y = x^3 - 3x^2 + 3x - 1 + 1 = (x-1)^3 + 1$

$\rightarrow y - 1 = (x-1)^3 \rightarrow x = \sqrt[3]{y-1} + 1 \rightarrow R_f = \mathbb{R}$

ب) $y = \dfrac{1}{x^2 - 2x} \rightarrow yx^2 - 2yx - 1 = 0 \rightarrow x = \dfrac{2y \pm \sqrt{4y^2 + 4y}}{2y}$ ① $\rightarrow y \neq 0$

$\rightarrow 4y^2 + 4y \geq 0 \rightarrow y^2 + y \geq 0 \rightarrow y(y+1) \geq 0$

	-1		0	
$+$	0	$-$	0	$+$

$\rightarrow$ ②

$① \cap ② \rightarrow R_y = (-\infty, -1] \cup (0, +\infty)$

۲)

الف) $y = x^2 - 4x + 10$

$\min \left| \begin{array}{l} \frac{-b}{2a} = \frac{4}{2} = 2 \\ 4 - 8 + 10 = 6 \end{array} \right. \rightarrow R_y = [6, +\infty)$

ب) $y = -x^2 + 6x + 3$

$\max \left| \begin{array}{l} \frac{-b}{2a} = \frac{-6}{-2} = 3 \\ -9 + 18 + 3 = 12 \end{array} \right. \rightarrow R_y = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3}$

$\min \left| \begin{array}{l} \frac{4}{2} = 2 \\ 4 - 8 - 3 = -7 \end{array} \right. \rightarrow \sqrt{[-7, +\infty)} = [0, +\infty)$

د) $y = \sqrt{6x - x^2}$

$\max \left| \begin{array}{l} \frac{-6}{-2} = 3 \\ -9 + 18 = 9 \end{array} \right. \rightarrow \sqrt{(-\infty, 9]} \rightarrow R_y = [0, 3]$

۳)

الف) $y = x^3 - 5x^2 + 2x + 1 \rightarrow R_y = \mathbb{R}$

ب) $y = x^7 - 6x^4 + 3x + 2 \rightarrow R_y = \mathbb{R}$

ج) $y = \sqrt{x^7 - 6x^2 + 5x + 1} \rightarrow \sqrt{(-\infty, +\infty)} = [0, +\infty)$

د) $y = (x^5 - 4x^2 + 3x + 1)^2 \rightarrow [(-\infty, +\infty)]^2 \rightarrow R_y = [0, +\infty)$

۴)

الف) $y = \dfrac{3x+1}{x-2}$ $R_y = \mathbb{R} - \{3\}$

ب) $y = \dfrac{2x+1}{x+1}$ $R_y = \mathbb{R} - \{2\}$

۵)

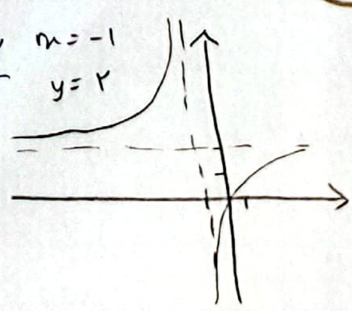
الف) $y = \sqrt{\dfrac{2x+4}{x+1}}$ $\sqrt{\mathbb{R} - \{2\}}$ $R_y = [0, +\infty) - \{\sqrt{2}\}$

ب) $y = \sqrt{\dfrac{3x+1}{x+2}}$ $\sqrt{\mathbb{R} - \{3\}} \rightarrow R_y = [0, +\infty) - \{\sqrt{3}\}$

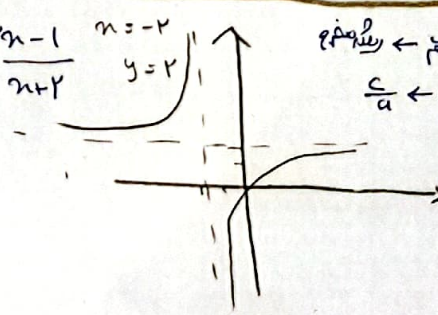
۶) مجانب قائم ← ریشه مخرج

مجانب افقی ← $\frac{c}{a}$

الف) $y = \dfrac{2x-3}{x+1}$ $x = -1$ $y = 2$



ب) $y = \dfrac{2x-1}{x+2}$ $x = -2$ $y = 2$



(۷)

الف) $y=\sin x+\frac{1}{\sin x} \to (-\infty,-2]\cup[2,+\infty)$

$-1\le \sin x\le 1 \leftarrow \sin x$

ب) $y=\frac{x^4+1}{x^3}=x^3+\frac{1}{x^3} \Rightarrow (-\infty,-2]\cup[2,+\infty)$

ج) $y=\frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}}$ $(-\infty,-2]\cup[2,+\infty)$

$\mathbb{R} \leftarrow \sqrt[3]{3} \Leftarrow \sqrt[3]{x}$

د) $\sqrt{x}+\frac{1}{\sqrt{x}} \to [2,+\infty)$

$\sqrt{x}\ge 0 \leftarrow \sqrt{x}$

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(۸)

الف) $y=x^2+\frac{1}{x^2+3}+3-3=x^2+3+\frac{1}{x^2+3}-3 \xrightarrow{x^2+3\ge 3} R_y=[\frac{10}{3}-3,+\infty)=[\frac{1}{3},+\infty)$

$x=0 \to \min=\frac{10}{3}$

ب) $y=\frac{x^2+5}{\sqrt{x^2+4}}=\sqrt{x^2+4}+\frac{1}{\sqrt{x^2+4}} \to R_y=[\frac{5}{2},+\infty)$

$\min \Rightarrow x=0 \to \frac{5}{2}$

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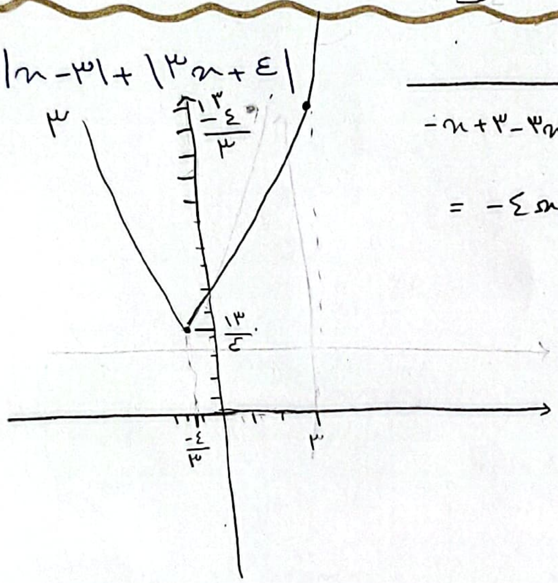
(۹)

$y=|x-3|+|3x+4|$



$x<-\frac{4}{3}$	$-\frac{4}{3}<x<3$	$x>3$
$-x+3-3x-4$	$-x+3+3x+4$	$x-3+3x+4$
$=-4x-1$	$=2x+7$	$=4x+1$

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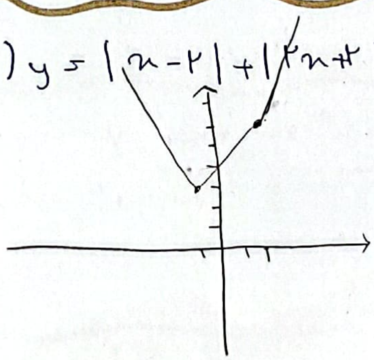
(۱۰)

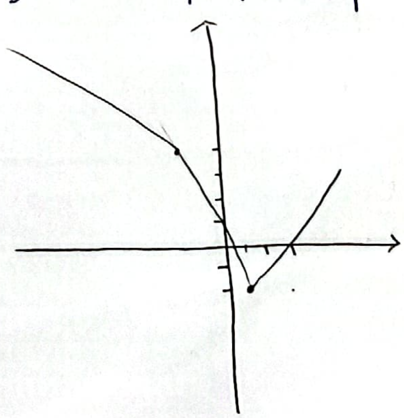
الف) $y=|x-2|+|2x+2|$



$x<-1$	$-1<x<2$	$x>2$
$-x+2-2x-2$	$-x+2+2x+2$	$x-2+2x+2$
$=-3x$	$=x+4$	$=3x$

$R_y=[3,+\infty)$

ب) $y=|2x-2|-|x+1|$



$x<-1$	$-1<x<1$	$x>1$
$-2x+2+x+1$	$-2x+2-x-1$	$2x-2-x-1$
$=-x+3$	$=-3x+1$	$=x-3$

$R_y=[-2,+\infty)$