~~السا~~ جلالی — تکلیف شماره ۶

۱- الف) $y = x^3 - 3x^2 + 3x + 1 - 1$ $\quad (x-1)^3 + 1 = y \quad (x-1)^3 = y-1$

$x - 1 = \sqrt[3]{y-1} \quad x = \sqrt[3]{y-1} + 1$

$R_f = \mathbb{R}$

ب) $y = \dfrac{1}{x^2 - 2x}$ $\quad yx^2 - 2yx = 1 \quad yx^2 - 2yx - 1 = 0$

$x = \dfrac{2y \pm \sqrt{4y^2 + 4y}}{2y}$

	-1		0	
$+$		$-$		$+$

$R_f = \mathbb{R} - (-1, 0]$

$4y^2 + 4y \geq 0 \quad y(y+1) \geq 0$

۲- الف) $y = x^2 - 4x + 10$ $\quad \min \begin{vmatrix} 2 \\ 6 \end{vmatrix}$ $\quad R_f = [6, +\infty)$

ب) $y = -x^2 + 6x + 3$ $\quad \max \begin{vmatrix} 3 \\ 12 \end{vmatrix}$ $\quad R_f = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3}$ $\quad \min \begin{vmatrix} 2 \\ -7 \end{vmatrix}$ $\Rightarrow [-7, +\infty) \xrightarrow{\sqrt{\ }} [0, +\infty)$

د) $y = \sqrt{-x^2 + 6x}$ $\quad \begin{vmatrix} 3 \\ 9 \end{vmatrix}$ $\quad (-\infty, 9] \xrightarrow{\sqrt{\ }} [0, 3]$

۳- الف) $y = x^3 - 5x^2 + 2x + 1$ $\quad \mathbb{R}$

ب) $y = x^7 - 6x^3 + 3x + 2$ $\quad \mathbb{R}$

ج) $y = \sqrt{x^7 - 6x^2 + 5x + 1}$ $\quad [0, +\infty)$

د) $y = (x^5 - 4x^2 + 3x + 1)^2$ $\quad [0, +\infty)$

۴- الف) $y = \dfrac{3x+1}{x-2}$ $\quad R_f = \mathbb{R} - \{3\}$

ب) $y = \dfrac{2x+1}{x+1}$ $\quad R_f = \mathbb{R} - \{2\}$

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$y = \sqrt{\frac{2x+2}{x+1}}$ $\quad \mathbb{R} - \{2\} \Rightarrow [0, +\infty) - \{\sqrt{2}\}$

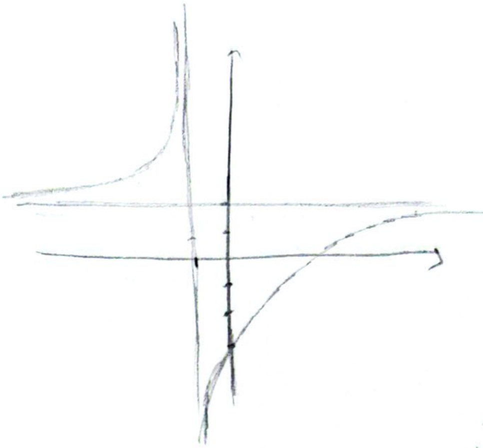
ب) $y = \sqrt{\frac{3x+1}{-x+3}}$ $\quad \mathbb{R} - \{-3\} \Rightarrow (0, +\infty)$

۶ -

الف) $y = \frac{2x-3}{x+1}$

مجانب قائم: -1

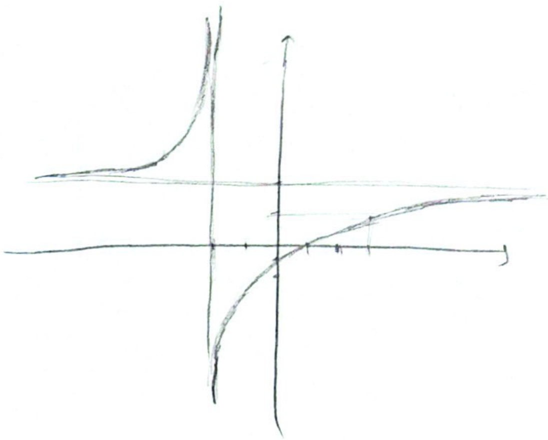
مجانب افقی: 2



ب) $y = \frac{2x-1}{x+2}$

مجانب قائم: -2

مجانب افقی: 2



۷ -

الف) $y = \sin x + \frac{1}{\sin x}$ $\quad R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^6+1}{x^3} = x^3 + \frac{1}{x^3}$ $\quad R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$ $\quad R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\quad R_f = [2, +\infty)$

۸ -

الف) $y = x^2 + \frac{1}{x^2+3} + 3 - 3 = \underbrace{x^2+3}_{\text{کمترین مقدار} = 3} + \frac{1}{x^2+3} - 3 \qquad 3 + \frac{1}{3} - 3$

$R_f = [\frac{1}{3}, +\infty)$

ب) $y = \dfrac{x^2+5}{\sqrt{x^2+4}} = \dfrac{x^2+4+1}{\sqrt{x^2+4}} = \underbrace{\sqrt{x^2+4}}_{\text{کمترین مقدار } = 2} + \dfrac{1}{\sqrt{x^2+4}}$ $\qquad 2+\dfrac{1}{2} = \dfrac{5}{2}$

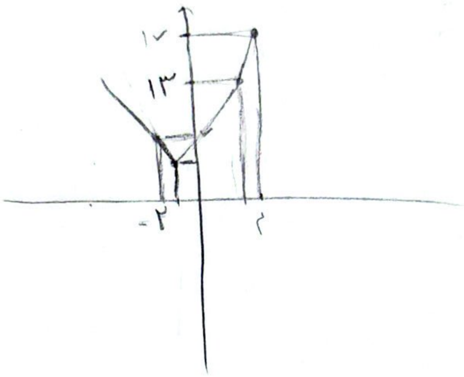
$R_f = [\frac{5}{2}, +\infty)$

۹-

$y = |x-3| + |3x+4|$

$-\frac{4}{3}$		3	
$-x+3-3x-4$ $-4x-1$	$-x+3+3x+4$ $2x+7$	$x-3+3x+4$ $4x+1$	

$\begin{vmatrix} 3 \\ 13 \end{vmatrix}$ $\quad \begin{vmatrix} -\frac{4}{3} \\ \frac{13}{3} \end{vmatrix}$ $\quad \begin{vmatrix} 4 \\ 17 \end{vmatrix}$ $\quad \begin{vmatrix} -2 \\ 7 \end{vmatrix}$

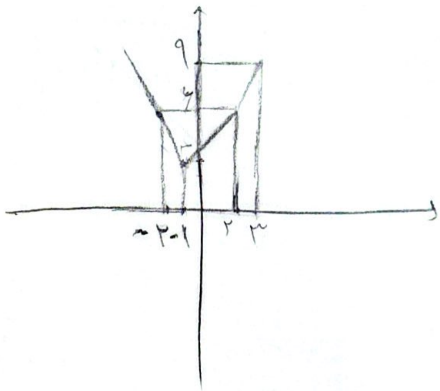


۱۰-

الف) $y = |x-2| + |2x+2|$

-1		2	
$-x+2-2x-2$ $-3x$	$-x+2+2x+2$ $x+4$	$x-2+2x+2$ $3x$	

$\begin{vmatrix} 2 \\ 6 \end{vmatrix}$ $\quad \begin{vmatrix} -2 \\ 6 \end{vmatrix}$ $\quad \begin{vmatrix} 3 \\ 9 \end{vmatrix}$ $\quad \begin{vmatrix} -1 \\ 3 \end{vmatrix}$

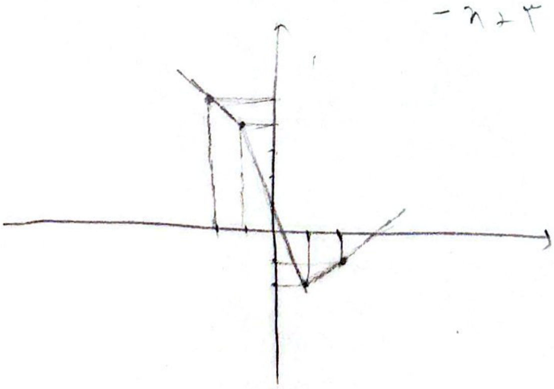


$R_f = [3, +\infty)$

ب) $y = |2x-2| - |x+1|$

-1		1	
$-2x+2+x+1$ $-x+3$	$-2x+2-x-1$ $-3x+1$	$2x-2-x-1$ $x-3$	

$\begin{vmatrix} 1 \\ -2 \end{vmatrix}$ $\quad \begin{vmatrix} -1 \\ 4 \end{vmatrix}$ $\quad \begin{vmatrix} 2 \\ -1 \end{vmatrix}$ $\quad \begin{vmatrix} -2 \\ 5 \end{vmatrix}$



$[-2, +\infty)$

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