السا جلالی — تکلیف شماره ۶

۱۹,۷۵

۱-

الف) $y=x^3-3x^2+3x+1-1$ $\quad (x-1)^3+1=y \quad (x-1)^3=y-1$

$x-1=\sqrt[3]{y-1} \quad x=\sqrt[3]{y-1}+1$

$R_f=\mathbb{R}$

ب) $y=\frac{1}{x^2-2x} \quad yx^2-2yx=1 \quad yx^2-2yx-1=0$

$x=\frac{2y\pm\sqrt{4y^2+4y}}{2y}$ $\quad$ $-1 \quad 0$; $+ \quad - \quad +$ $\quad R_f=\mathbb{R}-(-1,0]$

$4y^2+4y\geq 0 \quad y(y+1)\geq 0$

۲-

الف) $y=x^2-4x+10$ $\quad \min \begin{vmatrix} 2 \\ 6 \end{vmatrix} \quad R_f=[6,+\infty)$

ب) $y=-x^2+6x+3$ $\quad \max \begin{vmatrix} 3 \\ 12 \end{vmatrix} \quad R_f=(-\infty,12]$

ج) $y=\sqrt{x^2-4x-3}$ $\quad \min \begin{vmatrix} 2 \\ -7 \end{vmatrix} \Rightarrow [-7,+\infty) \xrightarrow{\sqrt{}} [0,+\infty)$

د) $y=\sqrt{-x^2+6x}$ $\quad \begin{vmatrix} 3 \\ 9 \end{vmatrix} \quad (-\infty,9] \xrightarrow{\sqrt{}} [0,3]$

۳-

الف) $y=x^3-5x^2+2x+1$ $\quad \mathbb{R}$

ب) $y=x^7-6x^3+2x+2$ $\quad \mathbb{R}$

ج) $y=\sqrt{x^7-6x^2+5x+1}$ $\quad [0,+\infty)$

د) $y=(x^5-4x^2+2x+1)^2$ $\quad [0,+\infty)$

۴-

الف) $y=\frac{3x+1}{x-2}$ $\quad R_f=\mathbb{R}-\{3\}$

ب) $y=\frac{2x+1}{x+1}$ $\quad R_f=\mathbb{R}-\{2\}$

۵- د

الف) $y = \sqrt{\frac{2x+2}{x+1}}$ $\quad \mathbb{R}-\{2\} \Rightarrow [0,+\infty)-\{\sqrt{2}\}$

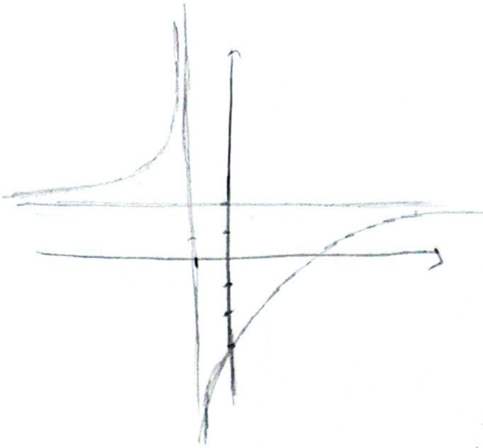
ب) $y = \sqrt{\frac{3x+1}{-x+2}}$ $\quad \mathbb{R}-\{-3\} \Rightarrow (0,+\infty)$

۶- د

الف) $y = \frac{2x-3}{x+1}$

مجانب قائم: -1

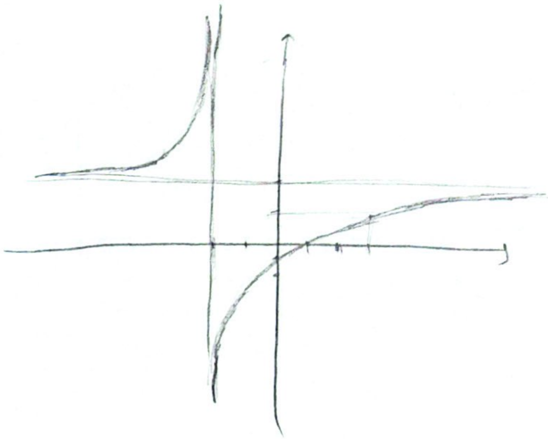
مجانب افقی: 2



ب) $y = \frac{2x-1}{x+2}$

مجانب قائم: -2

مجانب افقی: 2



۷- د

الف) $y = \sin x + \frac{1}{\sin x}$ $\quad R_f = (-\infty, -2] \cup [2, +\infty)$

ب) $y = \frac{x^4+1}{x^3} = x^3 + \frac{1}{x^3}$ $\quad R_f = (-\infty, -2] \cup [2, +\infty)$

ج) $y = \frac{\sqrt[3]{x^2}+1}{\sqrt[3]{x}} = \sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$ $\quad R_f = (-\infty, -2] \cup [2, +\infty)$

د) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ $\quad R_f = [2, +\infty)$

۸- د

الف) $y = x^2 + \frac{1}{x^2+3} + 3 - 3 = \underbrace{x^2+3}_{\text{کمترین مقدار} = 3} + \frac{1}{x^2+3} - 3 \quad 3 + \frac{1}{3} - 3$

$R_f = [\frac{1}{3}, +\infty)$

ب) $y = \frac{x^2+5}{\sqrt{x^2+4}} = \frac{x^2+4+1}{\sqrt{x^2+4}} = \underbrace{\sqrt{x^2+4}}_{\text{کمترین مقدار} = 2} + \frac{1}{\sqrt{x^2+4}}$ $\qquad 2+\frac{1}{2}=\frac{5}{2}$

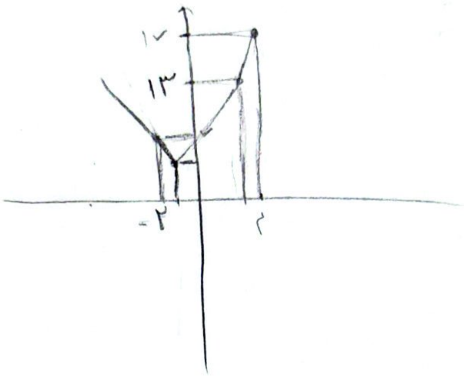
$R_f = [\frac{5}{2}, +\infty)$

۹- (۲)

$y = |x-3| + |3x+4|$

$-\frac{4}{3}$		3	
$-x+3-3x-4$	$-x+3+3x+4$	$x-3+3x+4$
$-4x-1$	$2x+7$	$4x+1$

x	y
3	13
$-\frac{4}{3}$	$\frac{13}{3}$
4	17
-2	7

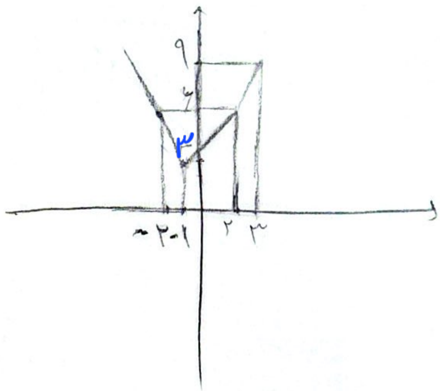


۱۰-

الف) $y = |x-2| + |2x+2|$ (۱٫۷۵)

-1		2	
$-x+2-2x-2$	$-x+2+2x+2$	$x-2+2x+2$
$-3x$	$x+4$	$3x$

x	y
2	6
-2	6
3	9
-1	3

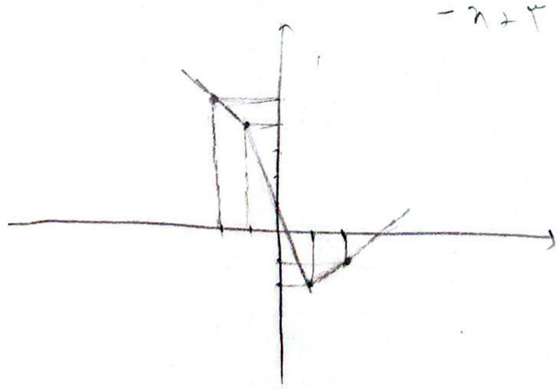


$R_f = [~~2~~ 3, +\infty)$

ب) $y = |2x-2| - |x+1|$

-1		1	
$-2x+2+x+1$	$-2x+2-x-1$	$2x-2-x-1$
$-x+3$	$-3x+1$	$x-3$

x	y
1	-2
-1	4
2	-1
-2	5



$[-2, +\infty)$