

(۱)

$$y = \sqrt{2^x f(x+1)} \quad f(x) = \left(\frac{1}{2}\right)^{x-1} - 1 \longrightarrow f(x+1) = \left(\frac{1}{2}\right)^x - 1$$

$$y = \sqrt{2^x \times \left(\left(\frac{1}{2}\right)^x - 1\right)} = \sqrt{2^x \times (2^{1-x} - 1)} \quad \frac{2^x \times (2^{1-x} - 1)}{x=0} \geq 0$$

$$2^{1-x} - 1 = 0 \rightarrow 2^{1-x} = 1 \rightarrow 1-x = 0 \rightarrow x = 1$$

$$\frac{0}{-} \quad \frac{1}{+}$$

$$D_y = [0, 1]$$

(۲)

$$f(x) = \sqrt{x + |x+2|} \longrightarrow f(-x) = \sqrt{-x + |-x+2|}$$

$$-x + |-x+2| \geq 0$$

$$-2x+2 \geq 0$$

$$2(-x+1) \geq 0 \rightarrow x = 1$$

$$\begin{array}{c|c} 2 & \\ \hline -x-x+2 & -x+x-2 \\ -2x+2 & -2 \end{array}$$

عدد منفی زیر را در یکال تعریف نمی شود

$$\frac{1}{+} \quad \frac{1}{-}$$

$$D_f = (-\infty, 1]$$

(۳)

$$y = \sqrt{\frac{x-1}{f(x)}}$$

$$x-1=0 \rightarrow x=1$$

$$f(x)=0 \rightarrow x=2, 3, -5$$

$$\frac{-5}{-} \quad \frac{1}{+} \quad \frac{2}{-} \quad \frac{3}{+}$$

$$D_y = (-5, 1] \cup (2, 3)$$

(۴)

$$y_1 = \frac{1}{9x^4 - \sqrt{x^2} - 5x - 5}$$

$$9x^4 - \sqrt{x^2} - 5x - 5 = 9x^4 - 3x^2 - x^2 - 5x - 5 + 1$$

$$\rightarrow (9x^4 - 4x^2 + 1) - x^2 - 5x - 4$$

$$\rightarrow (3x^2 - 1)^2 - (x+2)^2$$

$$(3x^2 - 1)^2 - (x+2)^2 \neq 0 \rightarrow (3x^2 - 1)^2 \neq (x+2)^2 \rightarrow 3x^2 - 1 \neq x+2$$

$$\rightarrow 3x^2 - x - 3 \neq 0$$

$$3x^2 - x - 3 = 3x^2 + ax + b \rightarrow a = -1, b = -3$$

$$y_2 = \frac{1}{3x^2 + ax + b}$$

$$\sqrt{-(a+b)} = \sqrt{-(-1-3)} = \sqrt{4} = 2$$

$$f(x) = x^5 + x$$

$$\begin{aligned} \Delta &= \sqrt{f(x) - f\left(\frac{x}{2}\right)} = \sqrt{x^5 + x - \frac{4x}{x^2} - \frac{x}{2}} = \sqrt{\left(x^5 - \frac{4x}{x^2}\right) + \left(x - \frac{x}{2}\right)} \\ &= \sqrt{\frac{1}{x^2}(x^5 - 4x) + \frac{1}{2}(x^5 - x)} = \sqrt{\frac{1}{x^2}(x^5 - x)(x^2 + 19 + 12x^2) + \frac{1}{2}(x^5 - x)} \\ &= \sqrt{\frac{1}{2}(x^5 - x) \left(\frac{1}{x^2}(x^5 + 19 + 12x^2) + 1 \right)} = \sqrt{\frac{1}{2}(x^5 - x) \left(x^3 + \frac{19}{x^2} + 12x + 1 \right)} \\ &\quad x^5 - x = 0 \rightarrow x^4 = 1 \rightarrow x = \pm 1 \\ &\quad \frac{1}{x} \rightarrow x = 0 \end{aligned}$$

$$\begin{array}{ccccccc} & -1 & & 0 & & 1 & \\ - & \phi & + & \frac{1}{\phi} & - & \phi & + \end{array}$$

$$D_2 = [-1, 0) \cup [1, +\infty)$$

$$\begin{aligned} f(x^5 + x) &= 1x^5 - 1 & x^5 + x &= 1 \rightarrow x = 1 \\ & & x^5 + x &= 1 \rightarrow x = 1 \end{aligned}$$

$$f(1) + f(1) = (1 \times 1) - 1 + (1 \times 1) - 1 = 1 + 1 = 2$$

$$\begin{aligned} f(x) &= x^5 - 4x^2 + 12x + 1 - 1 = (x-1)^5 + 1 \\ g(x) &= x^5 + 9x^2 + 12x + 12 - 12 = (x+3)^5 - 12 \end{aligned}$$

$$\frac{g(\sqrt[5]{12} - 1)}{f(\sqrt[5]{12} + 1)} = \frac{(\sqrt[5]{12} - 1 + 3)^5 - 12}{(\sqrt[5]{12} + 1 - 1)^5 + 1} = \frac{12 - 12}{1 + 1} = \frac{0}{2} = 0$$

$$f(x) = \sqrt{x + \epsilon \sqrt{x - \epsilon}} = \sqrt{x - \epsilon + \epsilon \sqrt{x - \epsilon} + \epsilon} = \sqrt{(\sqrt{x - \epsilon} + 1)^2} = \sqrt{x - \epsilon} + 1$$

$$g(x) = \sqrt{x - \epsilon \sqrt{x - \epsilon}} = \sqrt{x - \epsilon - \epsilon \sqrt{x - \epsilon} + \epsilon} = \sqrt{(\sqrt{x - \epsilon} - 1)^2} = \sqrt{x - \epsilon} - 1$$

$$f(x) + g(x) = \sqrt{x - \epsilon} + 1 + \sqrt{x - \epsilon} - 1 = 2\sqrt{x - \epsilon}$$

$$a + b\sqrt{x + c} = 2\sqrt{x - \epsilon} \rightarrow \begin{aligned} a &= 0 \\ b &= 2 \\ c &= -\epsilon \end{aligned} \quad \frac{a+b}{c} = \frac{0+2}{-\epsilon} = \frac{-2}{\epsilon}$$

$$f(z) = \frac{z+2}{z^2-4z+5}$$

$$z^2 - 4z + 5 = 0 \rightarrow (z-1)(z-3) = 0$$

$$\rightarrow z=1, z=3$$

$$D_f = \mathbb{R} - \{1, 3\}$$

$$g(z) = \{(2, 1), (1, 0), (3, 5), (0, 4)\}$$

$$D_f \cap D_g = \{2, 0\}$$

$$\frac{g}{f} = \{(2, \frac{-1}{3}), (0, 4)\}$$

$$الف) f(z) = \{(\frac{1}{2}, 2), (\frac{5}{2}, -1), (2, 2), (\frac{-1}{2}, 4)\}$$

$$ب) f(z) = \{(1, 2), (-1, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (2, 2), (-2, 2)\}$$

$$ج) z^2 g(z) + 1 = \{(-2, 19), (1, 3), (3, 9), (-1, 1)\}$$

$$د) \frac{zf}{g} = \{(1, -4), (3, -1), \underbrace{(-1, \frac{13}{9})}_{\text{تعريف نشده}}\} = \{(1, -4), (3, -1)\}$$