تکلیف شماره ی ۵ — هستی بابری — ۲۰ — یازدهم دختر C

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①

$y = \sqrt{x^3 f(x+1)}$ $\quad f(x) = \left(\frac{1}{2}\right)^{x-2} - 1 \longrightarrow f(x+1) = \left(\frac{1}{2}\right)^{x-1} - 1$

$y = \sqrt{x^3 \times \left(\left(\frac{1}{2}\right)^{x-1} - 1\right)} = \sqrt{x^3 \times (2^{1-x} - 1)}$ $\quad \underset{x=0}{x^3} \times (2^{1-x} - 1) \geq 0$ د

$2^{1-x} - 1 = 0 \to 2^{1-x} = 1 \to 1 - x = 0 \to x = 1$

$\begin{array}{ccccc} & 0 & & 1 & \\ - & \phi & + & \phi & - \end{array}$

$\boxed{D_y = [0, 1]}$

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②

$f(x) = \sqrt{x + |x+2|} \longrightarrow f(-x) = \sqrt{-x + |-x+2|}$ د

$-x + |-x+2| \geq 0$

2	
$-x - x + 2$ $-2x + 2$	$-x + x - 2$ -2 عدد منفی زیر رادیکال تعریف نمی شود

$-2x + 2 \geq 0$

$2(-x+1) \geq 0 \to x = 1$ $\quad \begin{array}{ccc} & 1 & \\ + & \phi & - \end{array}$

$\boxed{D_f = (-\infty, 1]}$

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③

$y = \sqrt{\dfrac{x-1}{f(x)}}$ $\quad x - 1 = 0 \to x = 1$

$f(x) = 0 \longrightarrow x = 2, 3, -4$ د

$\begin{array}{ccccccccc} & -4 & & 1 & & 2 & & 3 & \\ - & \not\phi & + & \phi & - & \not\phi & + & \not\phi & - \end{array}$

$\boxed{D_y = (-4, 1] \cup (2, 3)}$

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④

$y_1 = \dfrac{1}{9x^4 - 7x^2 - 4x - 3}$ $\quad 9x^4 - 7x^2 - 4x - 3 = 9x^4 - 6x^2 - x^2 - 4x - 4 + 1$

$\to (9x^4 - 6x^2 + 1) - x^2 - 4x - 4$

$\to (3x^2 - 1)^2 - (x+2)^2$ د

$(3x^2 - 1)^2 - (x+2)^2 \neq 0 \to (3x^2 - 1)^2 \neq (x+2)^2 \to 3x^2 - 1 \neq x + 2$

$\to 3x^2 - x - 3 \neq 0$

$3x^2 - x - 3 = 3x^2 + ax + b \to a = -1$, $\ b = -3$

$y_2 = \dfrac{1}{3x^2 + ax + b}$

$\sqrt{-(a+b)} = \sqrt{-(-3-1)} = \sqrt{4} = \boxed{2}$

(5)

$f(x)=x^3+x$

$y=\sqrt{f(x)-f(\frac{4}{x})}=\sqrt{x^3+x-\frac{64}{x^3}-\frac{4}{x}}=\sqrt{(x^3-\frac{64}{x^3})+(x-\frac{4}{x})}$

$=\sqrt{\frac{1}{x^3}(x^6-64)+\frac{1}{x}(x^2-4)}=\sqrt{\frac{1}{x^3}(x^2-4)(x^4+16+4x^2)+\frac{1}{x}(x^2-4)}$

$=\sqrt{\frac{1}{x}(x^2-4)\left(\frac{1}{x^2}(x^4+16+4x^2)+1\right)}=\sqrt{\frac{1}{x}(x^2-4)\underbrace{\left(x^2+\frac{16}{x^2}+5\right)}_{\text{همواره }+}}$

$x^2-4=0\to x^2=4\to x=\pm 2$

$\frac{1}{x}\to x=0$

	-2		0		2	
$-$	ϕ	$+$	$\not\exists$	$-$	ϕ	$+$

$D_y=[-2,0)\cup[2,+\infty)$

(6)

$f(x^3+x)=2x^2-1$

$x^3+x=2\to x=1$

$x^3+x=10\to x=2$

$f(2)+f(10)=(2\times 1)-1+(2\times 4)-1=1+7=8$

(7)

$f(x)=x^3-6x^2+12x+8-8=(x-2)^3+8$

$g(x)=x^3+9x^2+27x+27-27=(x+3)^3-27$

$\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=\frac{(\sqrt[3]{7}-3+3)^3-27}{(\sqrt[3]{2}+2-2)^3+8}=\frac{7-27}{2+8}=\frac{-20}{10}=-2$

(8)

$f(x)=\sqrt{x+4\sqrt{x-4}}=\sqrt{x-4+4\sqrt{x-4}+4}=\sqrt{(\sqrt{x-4}+2)^2}=\sqrt{x-4}+2$

$g(x)=\sqrt{x-4\sqrt{x-4}}=\sqrt{x-4-4\sqrt{x-4}+4}=\sqrt{(\sqrt{x-4}-2)^2}=\sqrt{x-4}-2$

$f(x)+g(x)=\sqrt{x-4}+2+\sqrt{x-4}-2=2\sqrt{x-4}$

$a+b\sqrt{x+c}=2\sqrt{x-4}\longrightarrow a=0,\ b=2,\ c=-4$

$\frac{a+b}{c}=\frac{0+2}{-4}=\frac{-1}{2}$

۹

$f(x) = \frac{x+2}{x^2-4x+3}$

$x^2 - 4x + 3 = 0 \rightarrow (x-1)(x-3) = 0$

$\rightarrow x = 1, x = 3$

$D_f = R - \{1, 3\}$

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$g(x) = \{(2,1), (1,0), (3,a), (0,4)\}$

$D_f \cap D_g = \{2, 0\}$

$\frac{g}{f} = \{(2, \frac{-1}{4}), (0, 4)\}$

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الف) $f(2x) = \{(\frac{1}{2}, 2), (\frac{3}{2}, -1), (2, 2), (\frac{-1}{2}, 4)\}$

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ب) $f(x^2) = \{(1,2)(-1,2)(\sqrt{3}, -1)(-\sqrt{3}, -1)(2,2)(-2,2)\}$

ج) $2g^2(x) + 1 = \{(-2, 19)(1, 3)(3, 9)(-1, 1)\}$

د) $\frac{2f}{g} = \{(1, -4)(3, -1)\underbrace{(-1, \frac{11}{5})}_{\text{تعریف نشده}}\} = \{(1, -4)(3, -1)\}$
