

1. $f(x) = (\frac{1}{x})^{x-1} - 1$ $y = \sqrt{x} f(x+1)$
 $f(x+1) \rightarrow \frac{1}{x+1}^{(x+1)-1} - 1 = \frac{1}{x+1} x - 1$
 $\rightarrow y = \sqrt{x} (\frac{1}{x+1})^{x-1} - 1 \rightarrow x^{\frac{1}{2}} (\frac{1}{x+1})^{x-1} - 1 \geq 0$
 $\frac{0}{-1} + \frac{1}{+1} - \rightarrow D_f = [0, 1]$

2. $f(x) = \sqrt{x+|x+1|}$
 $f(-x) = \sqrt{-x+|1-x|} \rightarrow -x+|1-x| \geq 0$
 $\rightarrow -x+x-1 \geq 0 \rightarrow -1 \geq 0 \times$
 $\rightarrow -x-x+1 \geq 0 \rightarrow x \leq 1$
 $D_f = (-\infty, 1]$

3. $y = \sqrt{\frac{x-1}{f(x)}} \rightarrow x-1 \geq 0 \rightarrow x \geq 1$ (I)
 $\frac{-1}{f(x)} - \frac{1}{f(x)} + \frac{1}{f(x)} + \frac{1}{f(x)} \rightarrow [-1, 1] \cup [1, +\infty)$ (II)
 $I \cap II = [1, 1] \cup [1, +\infty)$
 $\frac{-1}{1} - \frac{1}{1} + \frac{1}{1} + \frac{1}{1}$

4. $9x^2 - 12ax + 4a^2 - 3 = (3x^2 + ax + b)(3x^2 + cx + d)$
 $\rightarrow 9x^2 - 12ax + 4a^2 - 3 = 9x^2 + (3c+3a)x^2 + (3d+3ab)x + (3c+3a)x + 3d+3ab$
 $3c+3a=0 \quad c+a=0 \quad c=-a$
 $3d+3ab=-3 \quad 3d+3a(-a)=-3 \quad 3d-3a^2=-3$
 $d-a^2=-1 \quad d=a^2-1$
 $3d+3ab=-3 \quad 3(a^2-1)+3a(-a)=-3 \quad 3a^2-3-3a^2=-3$
 $-3=-3$
 $A = -1 \quad b = -a \quad \sqrt{a+b} = 1$

5. $f(x) = x^x + x \sqrt{f(x) - f(\frac{x}{2})} \rightarrow \sqrt{x^x + x \frac{4x}{x^2} - \frac{x}{x}}$
 $(x^x - x)(x^x + x) \geq 0$
 $x(x^x + 1) - \frac{x}{x} (\frac{14}{x^2} - 1) \geq 0$
 $D_f = [-1, 0] \cup [0, 1] \cup [1, +\infty)$

6. $f(x^x + x) < 12x^x - 1$
 $x=1 \rightarrow f(1) = 1(1) - 1 = 0$
 $x=1 \rightarrow f(1) = 1(1) - 1 = 0$
 $f(1) = 1(1) - 1 = 0$

7. $f(x) = x^x - 4x^x + 12x^x + (x-1)^x + 1$
 $g(x) = x^x + 9x^x + 12x^x = (a+x)^x - 12x^x$
 $\frac{g(\sqrt{x+1})}{f(\sqrt{x+1})} = \frac{(\sqrt{x+1})^x - 12x^x}{(\sqrt{x+1})^x + 1} = 1$

8. $\sqrt{x+1}\sqrt{x-1} + \sqrt{x-1}\sqrt{x-1}$
 $\sqrt{1\sqrt{x+1}\sqrt{x-1}} + \sqrt{x-1}\sqrt{x-1}$
 $\sqrt{x-1} + 1 + |\sqrt{x-1}-1|$
 $\frac{x-1}{x-1} \rightarrow \sqrt{x-1} + 1 + \sqrt{x-1}-1 = 2\sqrt{x-1}$
 $x > 1 \rightarrow \sqrt{x-1} + 1 - \sqrt{x-1} + 1 = 2$
 $a + b\sqrt{x+c} = \sqrt{x+c} \rightarrow a+b=1$
 $\frac{a+b}{c} = \frac{1}{x+c} = \frac{1}{x}$

9. $F(x) = \frac{x+1}{x^2-x+1} \rightarrow F(x) = [(1, 1), (1, 1)]$
 $(1, 1) \rightarrow (0, 1)$
 $\frac{g}{f} = [(1, 1), (1, 1)]$

10. (a) $[(\frac{1}{2}, 1), (\frac{1}{2}, -1), (1, 1), (-\frac{1}{2}, 1)]$
 (b) $[(\frac{1}{2}, 1), (\frac{1}{2}, -1), (1, 1), (-\frac{1}{2}, 1)]$
 (c) $[(1, 1), (1, 1), (1, 1), (-1, 1)]$
 (d) $[(1, 1), (1, 1), (1, 1), (-1, 1)]$

مطلوب