

1 $f(x) = (\frac{1}{x})^{x-1} - 1$ $y = \sqrt{x^{1/f(x)}}$
 $f(x+1) \rightarrow \frac{1}{x+1}^{(x+1)-1} - 1 = \frac{1}{x+1} x - 1$
 $\rightarrow y = \sqrt{x^{1/f(x)} (\frac{1}{x})^{x-1} - 1} \rightarrow x^{1/f(x)} (\frac{1}{x})^{x-1} - 1 \geq 0$
 $\frac{0}{-1} \frac{1}{+1} \rightarrow D_f = [0, 1]$

2 $f(x) = \sqrt{x+|x+1|}$
 $f(-x) = \sqrt{-x+|1-x|} \rightarrow -x+|1-x| \geq 0$
 $\rightarrow -x+x-1 \geq 0 \rightarrow -1 \geq 0 \times$
 $\rightarrow -x-x+1 \geq 0 \rightarrow x \leq 1$
 $D_f = (-\infty, 1]$

3 $y = \sqrt{\frac{x-1}{f(x)}} \rightarrow x-1 \geq 0 \rightarrow x \geq 1$
 $f(x) = \frac{-x}{-x+1} \rightarrow \frac{-x}{-x+1} \geq 0$
 $I \cap II = [1, 1] \cup [1, +\infty)$
 $D_f = (-1, 1] \cup (1, +\infty)$

4 $9x^2 - 12ax + 4a^2 - 3 = (3x^2 + ax + b)(3x^2 + cx + d)$
 $\rightarrow 9x^2 - 12ax + 4a^2 - 3 = 9x^2 + (3c+3a)x^2 + (3d+3a)x + (3c+3a)x + (3d+3a)x + (3c+3a)x + (3d+3a)x + (3c+3a)x + (3d+3a)x$
 $3c+3a=0 \quad c+a=0 \quad c=-a$
 $3d+3a=-3 \quad d+a=-1 \quad d=-1-a$
 $3(-1-a)+3a=-3 \quad -3-3a+3a=-3 \quad -3=-3$
 $D_f = [-1, +\infty)$

5 $f(x) = x^2 + x \sqrt{f(x) - f(\frac{1}{x})} \rightarrow \sqrt{x^2 + x \frac{4x^2 - 1}{x^2} - \frac{1}{x}}$
 $(x^2 - 1)(x^2 + 1) \geq 0$
 $x(x^2 + 1) - \frac{1}{x} (\frac{1}{x^2} - 1) \geq 0$
 $D_f = [-1, 1] \cup [0, 1] \cup [1, +\infty)$

6 $f(x^2 + 1) \leq 12x^2 - 1$
 $x=1 \rightarrow f(2) = 12(1) - 1 = 11$
 $x=2 \rightarrow f(5) = 12(4) - 1 = 47$
 $f(1) = 12(1) - 1 = 11$
 $f(4) = 12(16) - 1 = 191$

7 $f(x) = x^2 - 4x^2 + 12x + (x-1)^2 + 1$
 $g(x) = x^2 + 9x^2 + 12x = (x+1)^2 - 1$
 $\frac{g(\sqrt{x+1})}{f(\sqrt{x+1})} = \frac{(x+1)^2 - 1}{(x+1)^2 + 1}$

8 $\sqrt{x+1}\sqrt{x-1} + \sqrt{x-1}\sqrt{x+1}$
 $\sqrt{(x+1)(x-1)} + \sqrt{(x-1)(x+1)}$
 $\sqrt{x^2-1} + \sqrt{x^2-1}$
 $2\sqrt{x^2-1}$
 $\frac{x+1}{x-1} \geq \frac{1}{x-1}$
 $x+1 \geq 1$
 $x \geq 0$

9 $F(x) = \frac{x+1}{x^2-4x+4} \rightarrow F(x) = \frac{1}{x-2} + \frac{1}{x-2}$
 $(1, 2) \cup (2, 4)$
 $\frac{g}{f} = [1, 2) \cup (2, 4)$

10 $(1, 2) \cup (2, 4)$
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11 $(a^2 - 1)(a^2 + 1) \geq 0$
 $D_f = [-1, 1] \cup [1, +\infty)$