ترانه طاهریان، تکلیف شماره‌ی ۵، یازدهم دهنر C

$f(x+1) = \left(\frac{1}{2}\right)^{x+1-2} - 1 = \left(\frac{1}{2}\right)^{x-1} - 1$ ۱-

$y = \sqrt{x^3 f(x+1)} = \sqrt{x^3\left(\left(\frac{1}{2}\right)^{x-1} - 1\right)} \rightsquigarrow x^3 \times \left(\left(\frac{1}{2}\right)^{x-1} - 1\right) \geq 0$

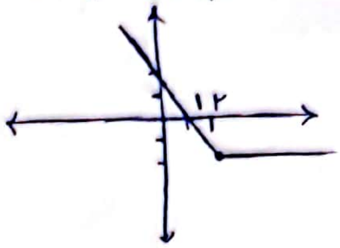
0 ← ، $\left(\frac{1}{2}\right)^{x-1} - 1 = 0 \Rightarrow \left(\frac{1}{2}\right)^{x-1} = 1 \Rightarrow$

$x - 1 = 0 \quad x = 1$

$\begin{array}{ccccc} & 0 & & 1 & \\ - & 0 & + & 0 & - \end{array} \Rightarrow D = [0, 1]$

$f(-x) = \sqrt{-x + |-x+2|} \rightsquigarrow -x + |2-x| \geq 0 \Rightarrow Df = (-\infty, 1]$ ۲-

$\begin{array}{ccc} & 2 & \\ -x+2-x = -2x+2 & & -x+x-2 = -2 \end{array}$



$y = \sqrt{\frac{x-1}{f(x)}} \rightsquigarrow \frac{x-1}{f(x)} \geq 0$ ۳-

	$-\infty$		-4		1		2		3
$x-1$		$-$		$-$	0	$+$		$+$	
$f(x)$		$+$	0	$-$		$-$	0	$+$	0
$\frac{x-1}{f(x)}$		$-$	$\nexists$	$+$	0	$-$	$\nexists$	$+$	$\nexists$

$Df = (-4, 1] \cup (2, 3)$

$y_1 = \frac{1}{9x^4 - 7x^2 - 4x - 3} \rightsquigarrow 9x^4 - 7x^2 - 4x - 3 \neq 0 \Rightarrow 9x^2 - 6x^2 + 1 - x^2 - 4x - 4 \neq 0$ ۴-

$\Rightarrow (3x^2-1)^2 - (x+2)^2 \neq 0 \Rightarrow (3x^2-1)^2 \neq (x+2)^2 \Rightarrow 3x^2 - 1 \neq x + 2 \Rightarrow$

$3x^2 - x - 3 \neq 0 \rightsquigarrow 3x^2 + ax + b \neq 0 \Rightarrow a = -1 \quad b = -3 \quad \boxed{\sqrt{-(a+b)} = 2}$

$y = \sqrt{f(x) - f\left(\frac{4}{x}\right)} = \sqrt{x^3 + x - \frac{64}{x^3} - \frac{4}{x}} \rightsquigarrow x^3 + x - \frac{64}{x^3} - \frac{4}{x} \geq 0$ ۵-

$\frac{x^6 + x^4 - 4x^2 - 64}{x^3} \geq 0 \quad x^6 + x^4 - 4x^2 - 64 \xrightarrow{x^2 = t} t^3 + t^2 - 4t - 64 =$

$\underbrace{t^3 - 64}_{(t-4)(t^2+4t+16)} + \underbrace{t^2 - 4t}_{t(t-4)} = (t-4)(t^2 + 5t + 16) \quad \hookrightarrow \Delta = 25 - 64 < 0$

← ریشه‌ی عبارت $t = 4$ است: $t = x^2 = 4 \Rightarrow x = \pm 2$

$\begin{array}{ccccccc} & -2 & & 0 & & 2 & \\ - & 0 & + & \nexists & - & 0 & + \end{array}$

$Df = [-2, 0) \cup [2, +\infty)$

$f(x^3 + x) = 2x^2 - 1 \rightsquigarrow f(2) \rightarrow x^3 + x = 2 \Rightarrow x^3 + x - 2 = 0$ ۶-

← جمع ضرایب صفر است ⇐ عبارت بر $x-1$ بخش‌پذیر است:

$\begin{array}{l|l} x^3 + x - 2 & x - 1 \\ \underline{-x^3 + x^2} & x^2 + x + 2 \\ x^2 + x - 2 & \\ \underline{-x^2 + x} & \\ 2x - 2 & \\ \underline{-2x + 2} & \\ 0 & \end{array}$

$\hookrightarrow \Delta = 1 - 8 = -7 < 0$

$\Rightarrow x = 1$

$\Rightarrow f(2) = 2 - 1 = \boxed{1}$

$\rightsquigarrow f(10) \rightsquigarrow x^3 + x = 10 \Rightarrow x^3 + x - 10 = 0$

$x^3 - 8 + x - 2 = (x-2)(x^2 + 2x + 4 + 1)$

$= (x-2)(x^2 + 2x + 5)$

$\hookrightarrow \Delta = 4 - 20 = -16 < 0$

$\Rightarrow x = 2 \Rightarrow f(10) = 2 \times 4 - 1 = \boxed{7}$

$f(2) + f(10) = 1 + 7 = 8$

۷-

$$f(x) = x^3 - 6x^2 + 12x - 8 + 8 = (x-2)^3 + 8$$

$$g(x) = x^3 + 9x^2 + 27x + 27 - 27 = (x+3)^3 - 27$$

$$\frac{g(\sqrt[3]{7} - 3)}{f(\sqrt[3]{2} + 2)} = \frac{(\sqrt[3]{7} - 3 + 3)^3 - 27}{(\sqrt[3]{2} + 2 - 2)^3 + 8} = \frac{-20}{10} = -2$$

۸-

$$f(x) = \sqrt{x + 4\sqrt{x-4} - 4 + 4} = \sqrt{x - 4 + 4\sqrt{x-4} + 4} = \sqrt{(\sqrt{x-4} + 2)^2} = |2 + \sqrt{x-4}| = \sqrt{x-4} + 2$$

همواره مثبت

$$g(x) = \sqrt{x - 4\sqrt{x-4} - 4 + 4} = \sqrt{x - 4 - 4\sqrt{x-4} + 4} = \sqrt{(\sqrt{x-4} - 2)^2} = |\sqrt{x-4} - 2|$$

$$f(x) + g(x) = \sqrt{x-4} + 2 + |\sqrt{x-4} - 2| \xrightarrow{x>8} \sqrt{x-4} + 2 + \sqrt{x-4} - 2 = 2\sqrt{x-4}$$

$c = -4 \quad b = 2 \quad a = 0$

$$\xrightarrow{x=8} \sqrt{x-4} + 2$$

$a = 2 \quad b = 1 \quad c = -4$

$$\xrightarrow{4 \leq x < 8} \sqrt{x-4} + 2 - \sqrt{x-4} + 2 = 4$$

$c \in R \quad b = 0 \quad a = 4$

$$\xrightarrow{x>8} \frac{a+b}{c} = \frac{2}{-4} = -\frac{1}{2}$$

$$\xrightarrow{x=8} \frac{a+b}{c} = \frac{-3}{4}$$

که نمی توان مقدار دقیقی برای $\frac{a+b}{c}$ تعیین کرد.

۹-

$$f(x) = \frac{x+2}{x^2 - 4x + 3}$$

$$g(x) = \{(2,1)(1,0)(3,5)(0,4)\}$$

$$f(2) = \frac{4}{4 - 8 + 3} = -4 \qquad f(3) = \frac{5}{9 - 12 + 3} = \frac{5}{0} \rightarrow \text{ت ن}$$

$$f(1) = \frac{3}{1 - 4 + 3} \rightarrow \text{ت ن} \qquad f(0) = \frac{2}{3}$$

$$\frac{g}{f} = \{(2, -\frac{1}{4})\cancel{(1, \frac{0}{\text{ت ن}})}\cancel{(3, \frac{5}{\text{ت ن}})}(0, \frac{4}{\frac{2}{3}} = 6)\} = \{(2, -\frac{1}{4})(0, 6)\}$$

۱۰-

الف) $f(2x) = \{(\frac{1}{2}, 2)(\frac{3}{2}, -1)(2, 2)(-\frac{1}{2}, 6)\}$

ب) $f(x^2) = \{(1, 2)(\sqrt{3}, -1)(2, 2)\}$

ج) $2g^2(x) + 1 = \{(-2, \underbrace{2 \times 9 + 1}_{19})(1, \underbrace{2 \times (-1)^2 + 1}_{3})(3, \underbrace{2 \times 4 + 1}_{9})(-1, \underbrace{2 \times 0 + 1}_{1})\}$

$= \{(-2, 19)(1, 3)(3, 9)(-1, 1)\}$

د) $\frac{2f}{g} = \{(1, \frac{4}{-1})\cancel{(-1, \frac{12}{0})}(3, \frac{-2}{2})\} = \{(1, -4)(3, -1)\}$