یازدهم دهم C-   19,5   فاطمه ثاقب - ۵

Date   No

1- $f(x)=\left(\frac{1}{2}\right)^{x-2}-1$ $y=\sqrt{x^3f(x+1)}$

$x^3f(x+1)\geq 0 \Rightarrow$

$x\geq 0,\ f(x+1)\geq 0 \to$

$f(x+1)=0 \Rightarrow \left(\frac{1}{2}\right)^{x-2+1}-1=0 \to \left(\frac{1}{2}\right)^{x-2+1}=1 \Rightarrow x=1$

$\Rightarrow$ 0 1 : $-\ +\ +\ -$ $\Rightarrow D_y=[0,1]$

2

2- $f(x)=\sqrt{x+|x+2|}$ $f(-x)$

$x+|x+2|\geq 0 \longrightarrow -x+|-x+2|\geq 0 \Rightarrow |2-x|\geq x$

$2-x\geq x \Rightarrow \boxed{x\leq 1} \Rightarrow D_f=(-\infty,1]$

$2-x\leq -x \Rightarrow 2\leq 0$ ✗

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3- $y=\sqrt{\frac{x-1}{f(x)}}$ $f(x)\neq 0 \Rightarrow x\neq\{-4,2,3\}$ $\frac{x-1}{f(x)}\geq 0 \longrightarrow$ $-4\ \ 1\ \ 2\ \ 3$: $-\ +\ -\ +\ -$

$\Rightarrow D_f=(-4,1]\cup(2,3)$

2

4- $y_1=\frac{1}{9x^4-7x^2-4x-3}$ $y_2=\frac{1}{3x^2+ax+b} \Rightarrow 3x^2+ax+b\neq 0 \to$

$x=\frac{-a\pm\sqrt{a^2-12b}}{6}$

$9x^4-7x^2-4x-3\neq 0 \Rightarrow \overbrace{(3x^2-x-3)}^{\Delta=37}\underbrace{(3x^2+x+1)}_{\Delta<0}$

$x=\frac{1\pm\sqrt{37}}{6} \longrightarrow \frac{1\pm\sqrt{37}}{6}=\frac{-a\pm\sqrt{a^2-12b}}{6}$

$\Rightarrow a=-1 \Rightarrow 1-12b=37 \Rightarrow b=-3 \Rightarrow \boxed{\sqrt{-(a+b)}=\sqrt{-(-4)}=2}$

2

5- $f(x)=x^3+x$ $y=\sqrt{f(x)-f(\frac{4}{x})} \Rightarrow y=\sqrt{x^3+x-\frac{64}{x^3}-\frac{4}{x}}=\sqrt{\frac{x^6+x^4-4x^2-64}{x^3}}$

$\longrightarrow x^6+x^4-4x^2-64\geq 0 \Rightarrow (x^4-4x^2)+\underbrace{(x^6-64)}_{(x^2)^3-(4)^3}\geq 0 \Rightarrow x^2(x^2-4)+(x^2-4)(x^4+4x^2+16)\geq 0 \Rightarrow$

$\Rightarrow (x^2-4)\underbrace{(x^4+5x^2+16)}_{\Delta<0,\ a>0 \to +\ \text{همواره}}\geq 0 \Rightarrow x^2-4\geq 0 \Rightarrow$ $-2\ \ 2$: $+\ -\ +$ $\Rightarrow D_y=(-\infty,-2]\cup[2,+\infty)$

1,5

6- $f(x^3+x)=2x^2-1$

$f(2)+f(10) \longrightarrow \xrightarrow{x=1} f(2)=1,\ \xrightarrow{x=2} f(10)=7 \Rightarrow \boxed{f(2)+f(10)=8}$

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7- $f(x)=x^3-6x^2+12x$ $g(x)=x^3+9x^2+27x$ $\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}$

$\sqrt[3]{7}=a \to g(a-3)=(a-3)^3+9(a-3)^2+27(a-3)=a^3-27 \to 7-27=-20$

$\sqrt[3]{2}=b \to f(b+2)=(b+2)^3-6(b+2)^2+12(b+2)=b^3+8 \to 2+8=10$

$\Rightarrow \boxed{\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=-2}$

2

8- $f(x)=\sqrt{x+4\sqrt{x-4}}$ $\qquad g(x)=\sqrt{x-4\sqrt{x-4}}$ $\qquad x\geq 4$

$\sqrt{x-4}=t \Rightarrow x=t^2+4 \longrightarrow f(x)=\sqrt{t^2+4+4\sqrt{t^2+4-4}}=\sqrt{t^2+4t+4}=\sqrt{(t+2)^2}=|t+2| \xrightarrow{t\geq 0} t+2$ $f(x)$
$\hookrightarrow t\geq 0$ $\qquad |t|=t$

$g(x)=\sqrt{t^2+4-4\sqrt{t^2+4-4}}=\sqrt{t^2-4t+4}=\sqrt{(t-2)^2}=|t-2|$

$f(x)+g(x)=t+2+|t-2|$

$\xrightarrow{t\geq 2} t+2+t-2=2t \longrightarrow x\geq 8 \longrightarrow 2t=2\sqrt{x-4}=a+b\sqrt{x+c}$
$\Downarrow$
$a=0,\ b=2,\ c=-4$

$\xrightarrow{t\leq 2} t+2-t+2=4 \longrightarrow 8\geq x\geq 4$

$\Rightarrow \dfrac{a+b}{c}=\dfrac{0+2}{-4}=-\dfrac{1}{2}$

9- $f(x)=\dfrac{x+2}{x^2-4x+3}$ $\qquad g(x)=\{(2,1)(1,0)(3,a)(0,4)\}$

$x^2-4x+3\neq 0 \longrightarrow (x-3)(x-1)\neq 0 \Rightarrow x\neq\{1,3\} \Rightarrow f(x)=\{\dots,(2,-4)\ (0,\tfrac{2}{3})\dots\}$

$\Rightarrow \dfrac{g}{f}=\{(2,-\tfrac{1}{4})(0,6)\}$

10- $f(x)=\{(1,2)(3,-1)(4,2)(-1,6)\}$
$g(x)=\{(1,-1)(3,2)(-2,3)(-1,0)\}$

الف) $f(2x)=\{(\tfrac{1}{2},2)(\tfrac{3}{2},-1)(2,2)(-\tfrac{1}{2},6)\}$

ب) $f(x^2)=\{(1,2)(-1,2)(\sqrt{3},-1)(-\sqrt{3},-1)(2,2)(-2,2)\}$

ج) $2g^2(x)+1=\{(1,3)(3,9)(-2,19)(-1,1)\}$

د) $\dfrac{2f}{g}=\{(1,-4)(3,-1)\cancel{(-1,\tfrac{12}{0})}\}=\{(1,-4)(3,-1)\}$
نیست

(ه

$$\frac{(x^{۲}-۴)(x^{۴}+۴x^{۲}+۱۶)}{x^{۲}} \geq ۰ \qquad D_f=[-۲,۰)\cup[۲,+\infty)$$

$$\begin{array}{ccccccc} & -۲ & & ۰ & & ۲ & \\ - & | & + & | & - & | & + \end{array}$$