DATE: SUB:

۲۰

$1\_\ f(x)=\left(\frac{1}{۲}\right)^{x-۲}-1 \qquad f(x+1)=\left(\frac{1}{۲}\right)^{x-1}-1=۲^{1-x}-1$ ۵

$y=\sqrt{x^۳ f(x+1)}=\sqrt{x^۳\left(۲^{1-x}-1\right)}$

$x^۳\left(۲^{1-x}-1\right)\geq ۰ \qquad \underset{-\quad ۰ \quad + \quad 1 \quad -}{\overline{\qquad\qquad}} \Rightarrow D_f=[۰,1]$

۵

$۲\_\ f(x)=\sqrt{x+|x+۲|}$

$f(-x)=\sqrt{-x+|۲-x|} \qquad |۲-x|-x\geq ۰ \qquad |۲-x|\geq x \rightarrow ۴+x^۲-۴x\geq x^۲$

$\rightarrow 1\geq x \qquad D_f=(-\infty,1]$ ۵

$۳\_\ y=\sqrt{\frac{x-1}{f(x)}} \qquad \underset{-\ \ -۴\ \ +\ \ 1\ \ -\ \ ۲\ \ +\ \ ۳}{\overline{\qquad\qquad\qquad}} \Rightarrow D_y=(-۴,1]\cup(۲,۳)$

$۴\_\ y_1=\frac{1}{۹x^۴-۷x^۲-۴x-۳}$ ۵

$۹x^۴-۷x^۲-۴x-۳=۹x^۴-۶x^۲+1-x^۲-۴x-۴=(۳x^۲-1)^۲-(x+۲)^۲=\underbrace{(۳x^۲+x+1)}_{\Delta<۰}\underbrace{(۳x^۲-x-۳)}_{\Delta>۰}$

$\rightarrow ۳x^۲-x-۳=۳x^۲+ax+b \rightarrow \begin{matrix} a=-1 \\ b=-۳ \end{matrix} \rightarrow \boxed{\sqrt{-(a+b)}=\sqrt{۴}=۲}$

$۵\_\ f(x)=x^۳+x$

$y=\sqrt{f(x)-f\left(\frac{۴}{x}\right)}=\sqrt{x^۳+x-\frac{۶۴}{x^۳}-\frac{۴}{x}}=\sqrt{\left(x-\frac{۴}{x}\right)\left(x^۲+۴+\frac{۱۶}{x^۲}\right)+\left(x-\frac{۴}{x}\right)}$

DATE: SUB:

$\Rightarrow y = \sqrt{\left(x-\frac{4}{x}\right)\left(x^2+5+\frac{16}{x^2}\right)} \geq 0 \Rightarrow$

-2		0		2	
$-$	$+$		$-$		$+$

$\Downarrow$

$D_f = [-2, 0) \cup [2, +\infty)$

$\underbrace{x^2+5+\frac{16}{x^2}}_{+ \text{تا} +}$ همواره مثبت

$x - \frac{4}{x} = \frac{x^2-4}{x} = \frac{(x+2)(x-2)}{x}$

6- $f(x^3+x) = 2x^2 - 1$

$x^3+x=10 \to x=2 \to f(10) = 8-1 = \boxed{7}$ $\quad f(10)+f(2) = \boxed{8}$

$x^3+x=2 \to x=1 \to f(2) = 2-1 = \boxed{1}$

7- $f(x) = x^3 - 6x^2 + 12x = (x-2)^3 + 8$

$g(x) = x^3 + 9x^2 + 27x = (x+3)^3 - 27$

$\longrightarrow \frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)} = \frac{7-27}{2+8} = \frac{-20}{10} = \boxed{-2}$

8- $f(x) = \sqrt{x+4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}+2)^2} = \sqrt{x-4}+2$

$g(x) = \sqrt{x-4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}-2|$

$f(x)+g(x) = a + b\sqrt{x+c}$

$\to 4 \le x \le 8 \to 2-\sqrt{x-4}+\sqrt{x-4}+2 = 4 \to a=4,\ b=0,\ c\in\mathbb{R}$

$\to x > 8 \to \sqrt{x-4}-2+\sqrt{x-4}+2 = 2\sqrt{x-4} \to b=2,\ a=0,\ c=-4 \to \frac{a+b}{c} = \boxed{-\frac{1}{2}}$

Elipon

DATE: SUB:

9- $f(x) = \dfrac{x+2}{x^2-4x+3} = \dfrac{x+2}{(x-1)(x-3)}$

$g(x) = \{(2,1),(1,0),(3,5),(0,4)\}$

$\dfrac{g}{f} = \{(2,-\frac{1}{4}),(0,6)\}$

10- $f(x) = \{(1,2),(3,-1),(4,2),(-1,6)\}$

$g(x) = \{(-2,3),(1,-1),(3,2),(-1,0)\}$

الف) $f(2x) = \{(\frac{1}{2},2),(\frac{3}{2},-1),(2,2),(-\frac{1}{2},6)\}$

ب) $f(x^2) = \{(1,2),(-1,2),(-\sqrt{3},-1),(\sqrt{3},-1),(2,2),(-2,2)\}$

ج) $2g^2(x)+1 = \{(-2,19),(1,3),(3,9),(-1,1)\}$

د) $\dfrac{2f}{g} = \{(1,-4),(3,-1)\}$