روژانو صدرالعلی — یازدهم ریاضی C — تکلیف ۵

۱- $f(x)=\left(\frac{1}{2}\right)^{x-2}-1$ $\qquad y=\sqrt{x^3 f(x+1)}\geq 0 \to$ دامنه؟

$\underbrace{x^3}_{0}\underbrace{f(x+1)}_{1}\geq 0$ $\qquad f(x+1)=\frac{1}{2}^{x+1-2}-1=\frac{1}{2}^{x-1}-1=0$ $\qquad \frac{1}{2}^{x-1}=1$ $\qquad 2^{-(x-1)}=2^0$

$-x+1=0$ $\qquad \boxed{x=1}$

0		1	
$-$	$+$		$-$

$\boxed{D_y=[0,1]}$ جواب

۲- $f(x)=\sqrt{x+|x+2|}$ $\qquad D_f \to f(-x)$

$f(-x)=\sqrt{-x+|2-x|}\geq 0$ $\qquad -x+|2-x|\geq 0$ $\qquad |2-x|\geq x$ $\qquad (2-x)^2\geq x^2$

$4+\cancel{x^2}-4x\geq \cancel{x^2}$ $\qquad 4\geq 4x$ $\qquad 1\geq x$

$\boxed{D_f=(-\infty,1]}$ جواب

۳- دامنه $\to y=\sqrt{\frac{x-1}{f(x)}}\geq 0$ $\qquad \frac{x-1}{f(x)}\geq 0$ $\qquad x-1 \to x=1$ $\qquad f(x) \to f(x)=0 \longrightarrow 2,3,-4$

-4		1		2		3	
$-$	$+$		$-$		$+$		$-$

$\boxed{D_f=(-4,1]\cup(2,3)}$ جواب

۴- $y_1=\frac{1}{9x^4-7x^2-4x-3}$ $\qquad y_2=\frac{1}{3x^2+ax+b}$ $\qquad D_{y_1}=D_{y_2}$ $\qquad \sqrt{-(a+b)}=?$

$9x^4-7x^2-4x-3=(9x^4-6x^2+1)-x^2-4x-4 \Rightarrow (3x^2-1)^2-(x+2)^2\neq 0$

$(3x^2-1)^2\neq(x+2)^2 \longrightarrow 3x^2-1\neq x+2 \Rightarrow 3x^2-x-3\neq 0$

$3x^2-x-3=3x^2+ax+b \Rightarrow a=-1,\ b=-3$

$\sqrt{-\underbrace{(-1-3)}_{4}}=\boxed{2}$ جواب

۵- $f(x)=x^3+x$ $\qquad y=\sqrt{f(x)-f\left(\frac{4}{x}\right)}\geq 0 \to$ دامنه؟

$\sqrt{x^3+x-\frac{64}{x^3}-\frac{4}{x}}$

$=\sqrt{\frac{1}{x^3}(x^6-64)+\frac{1}{x}(x^2-4)}$

$=\sqrt{\frac{1}{x^3}(x^2-4)(x^4+16+4x^2)+\frac{1}{x}(x^2-4)}$

$=\sqrt{\frac{1}{x}(x^2-4)\left(\frac{1}{x^2}(x^4+16+4x^2)+1\right)}$

$=\sqrt{\underbrace{\frac{1}{x}}_{0}\underbrace{(x^2-4)}_{\pm 2}\underbrace{\left(x^2+\frac{16}{x^2}+5\right)}_{\text{همواره مثبت}}}\geq 0$

-2		0		2	
$-$	$+$		$-$		$+$

$\boxed{D_y=[-2,0)\cup[2,+\infty)}$ جواب

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$f(x^3+x) = 2x^2-1 \qquad f(2)+f(10)=? \qquad \boxed{1+7=8}$ جواب

$x^3+x=2 \qquad x(x^2+1)=2 \to 1\times 2 \quad \hookrightarrow -1\times -2$

$\boxed{x=1} \quad 1(1+1)=2$

$x=-1 \quad -1(1+1) \Rightarrow -2$ غ ق ق

$f(2) = 2(1)^2-1 = \boxed{1}$

$x^3+x=10$

$x(x^2+1)=10 \to 2\times 5,\ -2\times -5,\ 1\times 10,\ -1\times -10$ } غ ق ق

$\boxed{x=2} \to 2(4+1)=10$

$f(10) = 2(2)^2-1 = \boxed{7}$

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$f(x) = x^3-6x^2+12x-8+8$

$(x-2)^3+8$

$f(\sqrt[3]{2}+2) \longrightarrow (\sqrt[3]{2}+\cancel{2}-\cancel{2})^3+8$

$10 = 2+8$

$g(x) = x^3+9x^2+27x+27-27$

$(x+3)^3-27$

$g(\sqrt[3]{7}-3) = (\sqrt[3]{7}\cancel{-3}\cancel{+3})^3-27$

$-20 = 7-27$

$\dfrac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)} = ? \qquad \dfrac{-20}{10} = \boxed{-2}$

جواب

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$f(x) = \sqrt{x+4\sqrt{x-4}}$

$2\times 2\times\sqrt{x-4}$

$(\sqrt{x-4}+2)^2$

$x\cancel{-4}\cancel{+4}+4\sqrt{x-4}$

$x+4\sqrt{x-4}$

$f(x) = \sqrt{(\sqrt{x-4}+2)^{\cancel{2}}} = |\sqrt{x-4}+2|$

$g(x) = \sqrt{x-4\sqrt{x-4}}$

$2\times 2\times\sqrt{x-4}$

$g(x) = \sqrt{(\sqrt{x-4}-2)^{\cancel{2}}}$

$g(x) = |\sqrt{x-4}-2|$

$f(x)+g(x) = a+b\sqrt{x+c}$

$\dfrac{a+b}{c} = ? \qquad \dfrac{0+2}{-4} = \boxed{\dfrac{-1}{2}}$

$|\sqrt{x-4}+2| + |\sqrt{x-4}-2|$

$= \sqrt{x-4}+2+\sqrt{x-4}+2 = 2\sqrt{x-4}$

$a+b\sqrt{x+c} = 2\sqrt{x-4}$

$a=0 \quad b=2 \quad c=-4$

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$f(x) = \dfrac{x+2}{x^2-4x+3}$

$(x-1)(x-3)$

$x \neq 1,\ 3 \qquad D_f = \mathbb{R}-\{1,3\}$

$g(x) = \{(2,1),\ \cancel{(1,5)},\ \cancel{(3,5)},\ (0,4)\}$

$\hookrightarrow f(2) = \dfrac{4}{(1)(-1)} = -4$

$\hookrightarrow f(0) = \dfrac{2}{(-1)(-3)} = \dfrac{2}{3}$

$\dfrac{g}{f} = ?$

$\{(2\ \tfrac{1}{-4})(0\ \tfrac{4}{\frac{2}{3}})\} \Rightarrow$

$\{(2,-\tfrac{1}{4})\ (0,6)\}$ جواب

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الف) $f(2x) = \{(\tfrac{1}{2},2)(\tfrac{3}{2},-1)(2,2)(-\tfrac{1}{2},6)\}$

ب) $f(x^2) = \{(\pm 1\ \ 2)(\pm\sqrt{3}\ -1)(\pm 2\ \ 2)\ \cancel{(\ \ 6)}\}$

ج) $2g^2(x)+1$

$\{(-2,19)\ (1,3)\ (3,9)\ (-1\ \ )\}$

د) $\dfrac{2f}{g} = \{(1\ \tfrac{4}{-1})(3\ \tfrac{-2}{2})\ \cancel{(-1\ \ )}\}$

$\{(1,-4)\ (3,-1)\}$