نام و نام خانوادگی ...... پاسخنامه تشریحی تکلیف شماره ۵ ..... کلاس ......

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$f(x) = \left(\frac{1}{2}\right)^{x-2}-1 \to f(x+1) = \left(\frac{1}{2}\right)^{x-1}-1$

$y = \sqrt{x^3 f(x+1)} = \sqrt{x^3(2^{1-x}-1)} \Rightarrow x^3(2^{1-x}-1) \geq 0$

$0 \quad 1$: $- \quad + \quad -$ $\Rightarrow \boxed{D = [0,1]}$

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$f(x) = \sqrt{x+|x+2|}$

$\Rightarrow f(-x) = \sqrt{-x+|-x+2|} \Rightarrow -x+|2-x| \geq 0 \to |2-x| \geq x$

$\to 2-x \geq x \to x \leq 1 \Rightarrow \boxed{D = (-\infty, 1]}$

$\to 2-x \leq -x \to 2 \leq 0$ ✗

۳

$y = \sqrt{\frac{x-1}{f(x)}} \Rightarrow \frac{x-1}{f(x)} \geq 0$

$-5 \quad -4 \quad 1 \quad 2 \quad 3$: $- \quad + \quad - \quad +$ $\Rightarrow \boxed{D_f = (-4,1] \cup (2,3)}$

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$y_1 = \frac{1}{9x^4-7x^2-4x-3} = \frac{1}{(9x^4-6x^2+1)-(x^2+4x+4)} = \frac{1}{(3x^2-1)^2-(x+2)^2}$

$\Rightarrow \frac{1}{(3x^2-x-3)(3x^2+x+1)} \to (3x^2-x-3)\underbrace{(3x^2+x+1)}_{\text{همواره مثبت}} \neq 0 \to 3x^2-x-3 \neq 0 \quad (1)$

$y_2 = \frac{1}{3x^2+ax+b} \to 3x^2+ax+b \neq 0 \quad (2)$

$\xrightarrow{(1),(2)} \begin{matrix} a=-1 \\ b=-3 \end{matrix} \Rightarrow \sqrt{-(a+b)} = \sqrt{-(-1+(-3))} = \boxed{2}$

۵

$f(x) = x^3+x$

$f\left(\frac{4}{x}\right) = \left(\frac{4}{x}\right)^3+\frac{4}{x}$

$\Rightarrow y = \sqrt{f(x)-f\left(\frac{4}{x}\right)} = \sqrt{x^3-\frac{64}{x^3}+x-\frac{4}{x}} = \sqrt{\left(x-\frac{4}{x}\right)\left(x^2+4+\frac{16}{x^2}\right)+\left(x-\frac{4}{x}\right)}$

$\Rightarrow \sqrt{\left(x-\frac{4}{x}\right)\left(x^2+\frac{16}{x^2}+5\right)} \to \sqrt{\left(\frac{x^2-4}{x}\right)\left(\frac{x^4+5x^2+16}{x^2}\right)} \to$ عبارت زیر رادیکال $\geq 0 \xrightarrow[\text{همواره مثبت}]{x^4+5x^2+16} \frac{x^2-4}{x} \geq 0$

$\to -2 \quad 0 \quad 2$: $- \quad + \quad - \quad +$ $\Rightarrow \boxed{D_f = [-2,0) \cup [2,+\infty)}$

$f(x^3+x) = 2x^2 - 1$

$\xrightarrow{x=1} f(2) = 2-1 = 1$

$\xrightarrow{x=2} f(10) = 2\times 4 - 1 = 7$ 6

$\boxed{f(2)+f(10) = 1+7 = 8}$

$f(x) = x^3 - 6x^2 + 12x \xrightarrow{+8-8} f(x) = (x-2)^3 + 8$

$g(x) = x^3 + 9x^2 + 27x \xrightarrow{+27-27} g(x) = (x+3)^3 - 27$

$$\left.\begin{aligned} f(\sqrt[3]{2}+2) &= (\sqrt[3]{2}+2-2)^3 + 8 = 10 \\ g(\sqrt[3]{7}-3) &= (\sqrt[3]{7}-3+3)^3 - 27 = -20 \end{aligned}\right\} \to \boxed{\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)} = -2}$$ 7

$f(x) = \sqrt{x+4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2| = \sqrt{x-4}+2$

$g(x) = \sqrt{x-4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2| \begin{cases} \xrightarrow{4\le x\le 8} 2-\sqrt{x-4} \\ \xrightarrow{x\ge 8} \sqrt{x-4}-2 \end{cases}$

$\xrightarrow{4\le x\le 8} f(x)+g(x) = 4+0\sqrt{x-4} \to \begin{matrix} a=4 \\ b=0 \\ c=-4 \end{matrix} \Rightarrow \frac{a+b}{c} = \frac{4+0}{-4} = \boxed{-1}$ 8

$\xrightarrow{x\ge 8} f(x)+g(x) = 0+2\sqrt{x-4} \to \begin{matrix} a=0 \\ b=2 \\ c=-4 \end{matrix} \Rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = \boxed{\frac{-1}{2}}$

$f(x) = \dfrac{x+2}{x^2-4x+3}$

$$\left.\begin{aligned} g(x) &= \{(2,1)(1,0)(3,5)(0,4)\} \\ f(2) &= \frac{2+2}{4-8+3} = -4 \\ f(1) &= \frac{1+2}{1-4+3} = \frac{3}{0} \text{ تن} \\ f(3) &= \frac{3+2}{9-12+3} = \frac{5}{0} \text{ تن} \\ f(0) &= \frac{0+2}{3} = \frac{2}{3} \end{aligned}\right\} \Rightarrow \boxed{\frac{g}{f}(x) = \left\{\left(2, \frac{-1}{4}\right)(0,6)\right\}}$$ 9

$f(x) = \{(1,2)(3,-1)(4,2)(-1,6)\}$

$g(x) = \{(-2,3)(1,-1)(3,2)(4,0)\}$

الف) $f(2x) = \left\{\left(\frac{1}{2},2\right)\left(\frac{3}{2},-1\right)(2,2)\left(\frac{-1}{2},6\right)\right\}$

ب) $f(x^2) = \{(1,2)(\sqrt{3},-1)(2,2)\}$ 10

ج) $2g^2(x)+1 = \{(-2,19)(1,3)(3,9)(-1,1)\}$

د) $\dfrac{2f}{g} = \{(1,-4)(3,-1)\}$