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نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ۵ کلاس ......[illegible]......

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$f(x)=(\frac{1}{2})^{x-2}-1 \longrightarrow f(x+1)=(\frac{1}{2})^{x-1}-1$

$y=\sqrt{x^3 f(x+1)}=\sqrt{x^3(2^{1-x}-1)} \Rightarrow x^3(2^{1-x}-1)\geq 0$

$\begin{array}{ccccc} & 0 & & 1 & \\ - & | & + & | & - \end{array} \Rightarrow \boxed{D=[0,1]}$

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$f(x)=\sqrt{x+|x+2|}$

$\Rightarrow f(-x)=\sqrt{-x+|-x+2|} \Rightarrow -x+|2-x|\geq 0 \rightarrow |2-x|\geq x$

$\longrightarrow 2-x\geq x \rightarrow x\leq 1 \quad \Rightarrow \boxed{D=(-\infty,1]}$

$\longrightarrow 2-x\leq -x \rightarrow 2\leq 0 \quad \times$

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$y=\sqrt{\frac{x-1}{f(x)}} \Rightarrow \frac{x-1}{f(x)}\geq 0$

$\begin{array}{ccccccccccc} & -5 & & -4 & & 1 & & 2 & & 3 & \\ & | & - & | & + & | & - & | & + & | & \end{array} \Rightarrow \boxed{D_f=(-4,1]\cup(2,3)}$

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$y_1=\frac{1}{9x^4-7x^2-4x-3}=\frac{1}{(9x^4-6x^2+1)-(x^2+4x+4)}=\frac{1}{(3x^2-1)^2-(x+2)^2}$

$\Rightarrow \frac{1}{(3x^2-x-3)(3x^2+x+1)} \rightarrow (3x^2-x-3)\underbrace{(3x^2+x+1)}_{\text{فاقد ریشه}}\neq 0 \rightarrow 3x^2-x-3\neq 0 \quad ①$

$y_2=\frac{1}{3x^2+ax+b} \longrightarrow 3x^2+ax+b\neq 0 \quad ②$

$\xrightarrow{①,②} \begin{array}{l} a=-1 \\ b=-3 \end{array} \Rightarrow \sqrt{-(a+b)}=\sqrt{-(-1+(-3))}=\boxed{2}$

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$f(x)=x^3+x$

$f(\frac{4}{x})=(\frac{4}{x})^3+\frac{4}{x}$

$\Rightarrow y=\sqrt{f(x)-f(\frac{4}{x})}=\sqrt{x^3-\frac{64}{x^3}+x-\frac{4}{x}}=\sqrt{(x-\frac{4}{x})(x^2+4+\frac{16}{x^2})+(x-\frac{4}{x})}$

$\Rightarrow \sqrt{(x-\frac{4}{x})(x^2+\frac{16}{x^2}+5)} \rightarrow \sqrt{(\frac{x^2-4}{x})(\frac{x^4+5x^2+16}{x^2})} \rightarrow$ عبارت زیر رادیکال $\geq 0 \xrightarrow[\text{همواره مثبت [illegible]}]{x^4+5x^2+16} \frac{x^2-4}{x}\geq 0$

$\rightarrow \begin{array}{ccccccc} & -2 & & 0 & & 2 & \\ - & | & + & | & - & | & + \end{array} \Rightarrow \boxed{D_f=[-2,0)\cup[2,+\infty)}$

$$f(x^3+x) = 2x^2-1$$

$$\xrightarrow{x=1} f(2) = 2-1 = 1$$

$$\xrightarrow{x=2} f(10) = 2\times 4-1 = 7$$

$$\boxed{f(2)+f(10) = 1+7 = 8}$$

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$$f(x) = x^3-6x^2+12x \xrightarrow{+8-8} f(x) = (x-2)^3+8$$

$$g(x) = x^3+9x^2+27x \xrightarrow{+27-27} g(x) = (x+3)^3-27$$

$$\left.\begin{array}{l} f(\sqrt[3]{2}+2) = (\sqrt[3]{2}+2-2)^3+8 = 10 \\ g(\sqrt[3]{7}-3) = (\sqrt[3]{7}-3+3)^3-27 = -20 \end{array}\right\} \rightarrow \boxed{\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)} = -2}$$

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$$f(x) = \sqrt{x+4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2| = \sqrt{x-4}+2$$

$$g(x) = \sqrt{x-4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2| \begin{array}{l} \xrightarrow{4\le x\le 8} 2-\sqrt{x-4} \\ \xrightarrow{x\ge 8} \sqrt{x-4}-2 \end{array}$$

$$\xrightarrow{4\le x\le 8} f(x)+g(x) = 4+0\sqrt{x-4} \rightarrow \begin{array}{l} a=4 \\ b=0 \\ c=-4 \end{array} \Rightarrow \frac{a+b}{c} = \frac{4+0}{-4} = \boxed{-1}$$

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$$\xrightarrow{x\ge 8} f(x)+g(x) = 0+2\sqrt{x-4} \rightarrow \begin{array}{l} a=0 \\ b=2 \\ c=-4 \end{array} \Rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = \boxed{\frac{-1}{2}}$$

$$f(x) = \frac{x+2}{x^2-4x+3}$$

$$\left.\begin{array}{l} g(x) = \{(2,1)(1,0)(3,5)(0,6)\} \\ f(2) = \frac{2+2}{4-8+3} = -4 \\ f(1) = \frac{1+2}{1-4+3} = \frac{3}{0} \text{ ت‌ن} \\ f(3) = \frac{3+2}{9-12+3} = \frac{5}{0} \text{ ت‌ن} \\ f(0) = \frac{0+2}{3} = \frac{2}{3} \end{array}\right\} \Rightarrow \boxed{\frac{g}{f}(x) = \{(2, \frac{-1}{4})(0,9)\}}$$

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$$f(x) = \{(1,2)(3,-1)(4,2)(-1,6)\}$$

$$g(x) = \{(-2,3)(1,-1)(3,2)(4,0)\}$$

الف) $f(2x) = \{(\frac{1}{2},2)(\frac{3}{2},-1)(2,2)(\frac{-1}{2},6)\}$

ب) $f(x^2) = \{(1,2)(\sqrt{3},-1)(2,2)\}$

۱۰ب) $\left\{\begin{array}{l} x^2=1 \rightarrow x=\pm 1 \\ x^2=3 \rightarrow x=\pm\sqrt{3} \\ x^2=4 \rightarrow x=\pm 2 \\ x^2=-1 \rightarrow \times \end{array}\right. \rightarrow \{(\pm 1,2)(\pm\sqrt{3},-1)(\pm 2,2)\}$

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ج) $3g^2(x)+1 = \{(-2,19)(1,3)(3,9)(-1,1)\}$

د) $\frac{2f}{g} = \{(1,-4)(3,-1)\}$