۲۰

نام و نام خانوادگی ....[illegible]..... پاسخنامه تشریحی تکلیف شماره ۵ ..... کلاس ....یازدهم ریاضی....

۱

$y = \sqrt{x^3 f(x+1)} \Rightarrow x^3 f(x+1) \geq 0 \to x^3\left(\left(\frac{1}{2}\right)^{x-1} - 1\right) \geq 0$

$f(x) = \left(\frac{1}{2}\right)^{x-2} - 1 \Rightarrow f(x+1) = \left(\frac{1}{2}\right)^{x-1} - 1$, $(x=0)$, $\left(\frac{1}{2}\right)^{x-1} - 1 = 0 \to \left(\frac{1}{2}\right)^{x-1} = 1$, $(x=1)$, $\left(\frac{1}{2}\right)^{0} = 1$

$\to$ جدول علامت: 0 , 1 : $- \; + \; -$ $\to D_y = [0, 1]$

۲ (نمره)

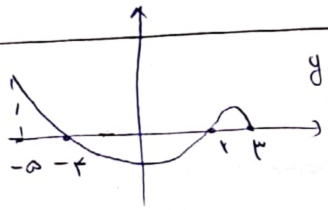
۲

$f(x) = \sqrt{x + |x+2|} \Rightarrow f(-x) = \sqrt{-x + |-x+2|} \Rightarrow -x + |-x+2| \geq 0 \to |-x+2| \geq x$

$\xrightarrow{\text{به توان ۲}} x^2 + 4 - 4x \geq x^2 \Rightarrow 4 \geq 4x \Rightarrow 1 \geq x \to D_f = (-\infty, 1]$

۲ (نمره)

۳



$y = \sqrt{\frac{x-1}{f(x)}} \to \frac{x-1}{f(x)} \geq 0$, $x = 1$; $x = 2, x = 3, x = -4$

$\to$ جدول علامت: $-5 \;\; -4 \;\; 1 \;\; 2 \;\; 3$: $- \; + \; - \; +$

$D_y = (-4, 1] \cup (2, 3)$

۲ (نمره)

۴

$y_1 = \frac{1}{9x^4 - 7x^2 - 4x - 3}$, $y_2 = \frac{1}{3x^2 + ax + b}$

$\to 9x^4 - 7x^2 - 4x - 3 = (3x^2 + ax + b)(3x^2 + cx + d) = 9x^4 + (3c + 3a)x^3 + (3d + ac + 3b)x^2 + (ad + bc)x + bd$

$\to \begin{cases} bd = -3 \xrightarrow{\text{مثلاً}} b = -3, \; d = 1 \\ ad + bc = -4 \to ad - ab = -4 \Rightarrow a(d - b) = -4 \to a = -1 \; (1 + (-3)) \\ 3c + 3a = 0 \to c = -a \\ 3d + ac + 3b = -7 \;\; 3 - 1 - 9 = -7 \checkmark \end{cases}$

$c = +1, \; a = -1, \; d = 1, \; b = -3$ آخر

$\to$ پس $\to 9x^4 - 7x^2 - 4x - 3 = (3x^2 + x + 1)(3x^2 - x - 3)$

ریشه ندارد $\Delta < 0$; $a = -1$, $b = -3$

$\sqrt{-(a+b)} = \sqrt{-(-4)} = \sqrt{4} = 2$; $-1 - 3$

۲ (نمره)

۵

$f(x) = x^3 + x \to f\left(\frac{4}{x}\right) = \frac{64}{x^3} + \frac{4}{x}$

$y = \sqrt{f(x) - f\left(\frac{4}{x}\right)} \Rightarrow f(x) - f\left(\frac{4}{x}\right) \geq 0 \Rightarrow x^3 + x - \frac{64}{x^3} - \frac{4}{x} \geq 0 \to x^3 - \frac{64}{x^3} + x - \frac{4}{x} \geq 0$

$\to \left(x - \frac{4}{x}\right)\left(x^2 + \frac{16}{x^2} + 4\right) + x - \frac{4}{x} \geq 0 \to \left(x - \frac{4}{x}\right)\left(x^2 + \frac{16}{x^2} + 4 + 1\right) \geq 0$

$\frac{x^2 - 4}{x} \neq 0 \to x = \pm 2, \; x = 0$; ضرب در x^2 $\to x^4 + 16 + 5x^2 = 0 \to \Delta < 0$ (-39) هیچ ریشه

$\to$ جدول علامت: $-2 \;\; 0 \;\; 2$: $- \; + \; - \; +$ $\to D_y = [-2, 0) \cup [2, +\infty)$

۲ (نمره)

۶

$f(x^3+x)=2x^2-1$

$x^3+x=2 \xrightarrow{x=1} f(2)=2-1=1$

$x^3+x=10 \xrightarrow{x=2} f(10)=2(4)-1=7$

$f(2)+f(10)=1+7=8$

د

۷

$f(x)=x^3-6x^2+12x=x^3-6x^2+12x-8+8=(x-2)^3+8$

$g(x)=x^3+9x^2+27x=x^3+9x^2+27x+27-27=(x+3)^3-27$

$g(\sqrt[3]{7}-3)=(\sqrt[3]{7}-3+3)^3-27=7-27=-20 \qquad f(\sqrt[3]{2}+2)=(\sqrt[3]{2}+2-2)^3+8=10$

$$\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=\frac{-20}{10}=-2$$

د

۸

$f(x)=\sqrt{x+4\sqrt{x-4}}$

$g(x)=\sqrt{x-4\sqrt{x-4}}$

$f(x)+g(x)=\sqrt{x+4\sqrt{x-4}}+\sqrt{x-4\sqrt{x-4}}=\sqrt{x-4+4\sqrt{x-4}+4}+\sqrt{x-4-4\sqrt{x-4}+4}$

$=\sqrt{(\sqrt{x-4}+2)^2}+\sqrt{(\sqrt{x-4}-2)^2}=|\sqrt{x-4}+2|+|\sqrt{x-4}-2|=a+b\sqrt{x+c}$

$8>x\geq 4$ → اولی نامنفی، دومی منفی → $\sqrt{x-4}+2+2-\sqrt{x-4}=4$

$x\geq 8$ → اولی نامنفی، دومی نامنفی → $\sqrt{x-4}+2+\sqrt{x-4}-2=2\sqrt{x-4}$ → $a=0,\ b=2,\ c=-4$

$$\frac{a+b}{c}=\frac{0+2}{-4}=-\frac{1}{2}$$

د

۹

$f(x)=\dfrac{x+2}{x^2-4x+3}$ $\qquad g(x)=\{(2,1)(1,0)(3,5)(0,4)\}$

$\dfrac{g}{f}=\left\{\left(2,-\dfrac{1}{4}\right),(0,6)\right\}$ $\qquad \left(\dfrac{4}{\frac{2}{3}}\right)$

$f(2)=\dfrac{2+2}{4-8+3}=\dfrac{4}{-1}=-4$ $\qquad f(0)=\dfrac{0+2}{0-0+3}=\dfrac{2}{3}$

$f(1)=\dfrac{1+2}{1-4+3}=\dfrac{3}{0}$ ✗

$f(3)=\dfrac{3+2}{9-12+3}=\dfrac{5}{0}$ ✗

د

۱۰

$f(x)=\{(1,2),(3,-1),(4,2),(-1,6)\}$ $\qquad g(x)=\{(-2,3)(1,-1)(3,2)(-1,0)\}$

الف) $f(2x)=\{(\frac{1}{2},2),(\frac{3}{2},-1),(2,2),(-\frac{1}{2},6)\}$

$2x=1\to x=\frac{1}{2},\ 2x=3\to x=\frac{3}{2},\ 2x=4\to x=2,\ 2x=-1\to x=-\frac{1}{2}$

ب) $f(x^2)=\{(1,2),(-1,2),(\sqrt{3},-1),(-\sqrt{3},-1),(2,2),(-2,2)\}$

$x^2=1\to x=\pm1,\ x^2=3\to x=\pm\sqrt{3},\ x^2=4\to x=\pm2,\ x^2=-1$ ✗

ج) $2g^2(x)+1=\{(-2,19),(1,3),(3,9),(-1,1)\}$

$g^2(x)=\{(-2,9),(1,1),(3,4),(-1,0)\}$

$2g^2(x)=\{(-2,18),(1,2),(3,8),(-1,0)\}$

د) $\dfrac{2f}{g}=\{(1,-4),(3,-1),\sout{(-1,\frac{12}{0})}\}$ تقاطع ۳ تا

$2f=\{(1,4),(3,-2),(4,4),(-1,12)\}$

د