۵

Subject :   دریا دیانی درودسنی

Year.   Month.   Date.   ( )

۱-

$f(x)=\left(\frac{1}{2}\right)^{x-2}-1$

$y=\sqrt{x^2 f(x+1)}$

$x^2 f(x+1)\geq 0 \rightarrow x^2\times\left(\left(\frac{1}{2}\right)^{x-1}-1\right)\geq 0$

$x^2=0 \rightarrow x=0$

$\left(\frac{1}{2}\right)^{x-1}-1=0 \rightarrow x=1$

$-\ \ 0\ +\ 1\ -\ \rightarrow D_f=[0\ \ 1]$

۲-

$f(x)=\sqrt{x+|x+2|}$ $\qquad f(-x)=\sqrt{-x+|-x+2|}$

۱) $x<2 \rightarrow -x-x+2=-2x+2\geq 0 \rightarrow \boxed{x\leq 1}$

$D_f=(-\infty\ \ 1]$

۲) $x>2 \rightarrow -x-(-x+2)=-x+x-2=-2\geq 0$ غ ق ق

۳-

$y=\sqrt{\frac{x-1}{f(x)}}$

۱) $f(x)\neq 0 \rightarrow x\neq 3, 2, -4$

۲) $\frac{x-1}{f(x)}\geq 0$ $\qquad -4\quad 1\quad 2\quad 3$ $\qquad -\ +\ -\ +\ -$

$D_f=(-4\ \ 1]\cup(2\ \ 3)$

۴- ۴

$D_{y_1}=D_{y_2}$

$\sqrt{-(a+b)}=?\rightarrow\sqrt{-(-4)}=2$

$y_1=\frac{1}{9x^4-6x^3-x^2-4x-4}\rightarrow 9x^4-6x^3-x^2-4x-4+1$

$(9x^4-6x^3+1)-x^2-4x-4$

$(3x^2-1)^2-(x+2)^2\neq 0\rightarrow 3x^2-1=x+2$

$y_2=\frac{1}{3x^2+ax+b}$

$3x^2-x-3=3x^2+ax+b \longleftarrow 3x^2-x-3=0$

$b=-3 \qquad a=-1$

۵-

$f(x)=x^3+x$

$y=\sqrt{f(x)-f(\frac{4}{x})}\Rightarrow f(x)-f(\frac{4}{x})\geq 0\rightarrow x^3+x-\left(\frac{64}{x^3}+\frac{4}{x}\right)=x^3+x-\frac{64}{x^3}-\frac{4}{x}$

$x^3-\frac{64}{x^3}+x-\frac{4}{x}=\left(x-\frac{4}{x}\right)\left(x^2+\frac{16}{x^2}+4\right)+\left(x-\frac{4}{x}\right)=\left(x-\frac{4}{x}\right)\underbrace{\left(x^2+\frac{16}{x^2}+5\right)}_{\text{همواره مثبت}}\geq 0$

حل دوم

$\frac{x^2-4}{x}\geq 0 \rightarrow$ $\qquad -2\quad 0\quad 2$ $\qquad -\ +\ -\ +$ $\qquad D_f=[-2\ \ 0)\cup[2\ \ +\infty)$

۶-

$f(x^2+x)=2x^2-1$

$f(2)\rightarrow x^2+x=2\rightarrow x^2+x-2 \rightarrow + \qquad f(10)\rightarrow x^3+x=10\rightarrow x^3+x-10=0$

$x^2+x-2\ |\ x-1 \longrightarrow x=1$

$-x^2+x^2 \qquad x^2+x+2$

x^2+x-2 $\qquad$ صفر ندارد $\qquad 2\times(1)^2-1=1$

$-x^2+x$

$2x-2$

$x^3-8+(x-2)$

$(x-2)(x^2+4+2x)+(x-2)$

$(x-2)\underbrace{(x^2+2x+5)}_{\text{صفر ندارد}}=0\rightarrow x=2$

$f(2)+f(10)=1+7=8$ $\qquad 2\times(2)^2-1=7$

PAPCO

جواب سوال

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$f(x)=x^3-6x^2+12x \xrightarrow{+8} f(x)=(x-2)^3+8$

$(x-2)^3=x^3-6x^2+12x-8$

$g(x)=x^3+9x^2+27x \xrightarrow{-27} g(x)=(x+3)^3-27$

$(x+3)^3=x^3+9x^2+27x+27$

$\dfrac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=\dfrac{7-27}{2+8} \rightarrow \dfrac{-20}{10}=\boxed{-2}$

(۸)

$f(x)=\sqrt{x+4\sqrt{x-4}} \longrightarrow \sqrt{x-4+4\sqrt{x-4}+4}=\sqrt{(\sqrt{x-4}+2)^2}$

$g(x)=\sqrt{x-4\sqrt{x-4}} \longrightarrow \sqrt{x-4-4\sqrt{x-4}+4}=\sqrt{(\sqrt{x-4}-2)^2}$ D_f

$f(x)=\sqrt{x-4}+2$

$g(x)=|\sqrt{x-4}-2|$

$f(x)+g(x)=a+b\sqrt{x+c} \longrightarrow \sqrt{x-4}+2+\sqrt{x-4}-2=2\sqrt{x-4}$ $\begin{cases}a=0\\ b=2\\ c=-4\end{cases}$

$\dfrac{a+b}{c}=\dfrac{0+2}{-4}=\boxed{-\dfrac{1}{2}}$

(۹)

$f(x)=\dfrac{x+2}{x^2-4x+3} \longrightarrow \dfrac{x+2}{(x-3)(x-1)} \rightarrow D_f=R-\{1,3\}$

$g(x)=\{(2,1)\ (1,5)\ (3,5)\ (0,4)\}$

$\dfrac{g}{f}=? \rightarrow \{(2,\dfrac{1}{-4})\ (0,\dfrac{4}{\frac{2}{3}})\} \longrightarrow \{(2,\dfrac{-1}{4}),(0,6)\}$

(۱۰)

$f(x)=\{(1,2),(3,-1),(4,2),(-1,6)\}$

$g(x)=\{(-2,3),(1,-1),(3,2),(-1,0)\}$

الف) $f(2x) \rightarrow 2x=1 \rightarrow x=\frac{1}{2}$ $\quad 2x=3 \rightarrow x=\frac{3}{2}$ $\quad 2x=4 \rightarrow x=2$ $\quad 2x=-1 \rightarrow x=-\frac{1}{2}$

$f(2x)=\{(\frac{1}{2},2)(\frac{3}{2},-1),(2,2),(-\frac{1}{2},6)\}$

ب) $f(x^2) \rightarrow x^2=1 \rightarrow x=\pm 1$ $\quad x^2=3 \rightarrow x=\pm\sqrt{3}$ $\quad x^2=4 \rightarrow x=\pm 2$ $\quad x^2=-1 \rightarrow$ غ ق ق

$f(x^2)=\{(\pm 1,2),(\pm\sqrt{3},-1)(\pm 2,2)\}$

ج) $2g^2(x)+1=\{(-2\quad 19)(1\quad 3)(3\quad 9)(-1\quad 1)\}$

د) $\dfrac{2f}{g}=\{(1\quad -4)\ (3\quad -1)\ \cancel{(-1\quad \frac{12}{0})}\}$ غ ق ق $\longrightarrow \{(1,-4)(3,-1)\}$

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