1) $y=\sqrt{x^3 f(x+1)} \to \sqrt{x^3\times\left(\left(\frac{1}{2}\right)^{x-1}-1\right)}$; $x-1=0 \to x=1$

$D_f=[0,1]$

0		1	
−	+		−

2) $f(-x)=\sqrt{-x+|-x+2|} \to -x+|-x+2|\geq 0 \to -x\geq -|x+2| \to x\leq |x+2|$

$x\leq -x+2 \to 2x\leq 2 \to \boxed{x\leq 1}$ − $x\geq x-2 \to 0\geq 2$ ∨ $D_f=(-\infty,1]$

3) $y=\sqrt{\frac{x-1}{f(x)}}$

−4		1		2		3
−	+		−		+	

$\to D_f=(-4,1]\cup(2,3)$

4) $9x^4-7x^2-4x-3 \;\big|\; 3x^2+ax+b$

$-9x^4 \mp 3ax^3 \mp 3bx^2 \qquad 3x^2-ax+1$

$-3ax^3-7x^2+3bx^2-4x-3$

$\cancel{3ax^3} \pm a^2x^2 \pm abx$

$x^2(a^2-7+3b)+x(ab-4)-3=0$

$\pm 3x^2 \pm ax \pm b$

0

$\to \boxed{b=-3}$ $a^2-3b-7=3 \to$

$a^2+9-7=3 \to a^2=1 \to a=\pm 1$

$a=ab-4 \Rightarrow a=-3a-4 \Rightarrow -4a=+4$

$\boxed{a=-1}$

$\sqrt{-(a+b)}=\sqrt{-(-1-3)} \to \sqrt{4}=\boxed{2}$

5) $f(x)=x^3+x \quad f\left(\frac{4}{x}\right)=\frac{4^3}{x^3}+\frac{4}{x}$

$y=\sqrt{f(x)-f\left(\frac{4}{x}\right)} \to f(x)-f\left(\frac{4}{x}\right)\geq 0 \quad x^3+x-\frac{4^3}{x^3}-\frac{4}{x}\geq 0$

$\frac{x^6+x^4-4x^2-64}{x^3}\geq 0 \xrightarrow{x^2=z} \frac{z^3+z^2-4z-64}{x^3} \to \frac{(z-4)(z^2+5z+16)}{x^3}$

$z-4=0 \to x^2=4 \to x=\pm 2$

−2		0		2	
−	+	−		+	

$\to \boxed{D_f=[-2,0)\cup[2,+\infty)}$

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۶) $f(x^3+x)=2 \to x^3+x=2 \Rightarrow x^3+x-2=0 \to x^3+x-2 \,\big|\, x-1 \;,\; x^2+x+2 \to (x-1)(x^2+x+2)=0$

$\xrightarrow{x=1} f(2)=1$

$x^3+x=10 \;\; x^3+x-10=0 \;\; x^3+x-10 \,\big|\, x-2 \;,\; x^2+2x+5 \to (x-2)(x^2+2x+5)=0$

$\xrightarrow{x=2} f(10)=7 \to 1+7=\boxed{8}$

$f(x)=x^3-6x^2+12x=(x-2)^3+8 \to (\sqrt[3]{2}+2-2)^3+8 \to 2+8=10$

۷) $g(x)=x^3+9x^2+27x=(x+3)^3-27 \to (\sqrt[3]{7}-3+3)^3-27 \to 7-27=-20$

$$\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=\frac{-20}{10}=\boxed{-2}$$

$z=\sqrt{x-4} \to x-4\geq 0 \to x\geq 4$

۸) $x\geq 4 \to z\geq 0 \to x=z^2+4$

$f(x)=\sqrt{z^2+4z+4} \to \sqrt{(z+2)^2} \to z+2$

$g(x)=\sqrt{z^2-4z+4} \to \sqrt{(z-2)^2} \to |z-2|$

$f(x)+g(x)=2z=2\sqrt{x-4} \to a+b\sqrt{x+c} \to a=0 \;\; b=2 \;\; c=-4$

$\frac{a+b}{c}=\frac{2}{-4}=\boxed{-\frac{1}{2}}$

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۹) $f(x)=\frac{x+2}{(x-1)(x-3)} \to R-\{1,3\}$ ، $\frac{g}{f}=\{(2,-\frac{1}{4}),(0,9)\}$

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۱۰)

الف) $f(2x)=\{(\frac{1}{2},2),(\frac{3}{2},-1),(2,1),(-\frac{1}{2},9)\}$

ب) $f(x^2)=\{(\pm 1,2),(\pm\sqrt{3},-1),(\pm 2,1)\}$

ج) $2g^2(x)+1=\{(-2,19),(1,3),(3,9),(-1,1)\}$

د) $\frac{2f}{g}=\{(1,-4),(3,-1)\}$