شهرزاد شوشتری ۵ (۲۰)

۱) (۲)

$y = \sqrt{x^3 f(x+1)} \to \sqrt{x^3 \times ((\frac{1}{2})^{x-1} - 1)}$; $x-1=0 \to x=1$

$D_f = [0, 1]$

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۲) (۲)

$f(-x) = \sqrt{-x + |-x+2|} \to -x + |-x+2| \geq 0 \to -x \geq -|x+2| \to x \leq |x+2|$

$x \leq -x+2 \to 2x \leq 2 \to \boxed{x \leq 1}$ — $x \geq x-2 \to 0 \geq 2$ ✓ $D_f = (-\infty, 1]$

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۳) (۲)

$y = \sqrt{\frac{x-1}{f(x)}}$ $\to D_f = (-4, 1] \cup (2, 3)$

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۴) (۲)

$9x^4 - 7x^2 - 4x - 3 \mid 3x^2 + ax + b$

$-9x^4 + 3ax^3 + 3bx^2 \quad 3x^2 - ax + 1$

$-3ax^3 - 7x^2 + 3bx^2 - 4x - 3$

$-3ax^3 - a^2x^2 - abx$

$x^2(a^2 - 7 + 3b) + x(ab - 4) - 3 = 0$

$+3x^2 + ax + b$

$\to \boxed{b = -3}$ $a^2 + 3b - 7 = 3 \to$

$a^2 + 9 - 7 = 3 \to a^2 = 1 \to a = \pm 1$

$a = ab - 4 \to a = -3a - 4 \to -4a = 4$

$\boxed{a = -1}$

$\sqrt{-(a+b)} = \sqrt{-(-1-3)} \to \sqrt{4} = \boxed{2}$

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۵) (۲)

$f(x) = x^3 + x \quad f(\frac{4}{x}) = \frac{4^3}{x^3} + \frac{4}{x}$

$y = \sqrt{f(x) - f(\frac{4}{x})} \to f(x) - f(\frac{4}{x}) \geq 0 \quad x^3 + x - \frac{4^3}{x^3} - \frac{4}{x} \geq 0$

$\frac{x^6 + x^4 - 4x^2 - 64}{x^3} \geq 0 \xrightarrow{x^2 = z} \frac{z^3 + z^2 - 4z - 64}{x^3} \to \frac{(z-4)(z^2 + 5z + 16)}{x^3}$

$z - 4 = 0 \to x^2 = 4 \to x = \pm 2 \to$

$\to \boxed{D_f = [-2, 0) \cup [2, +\infty)}$

۶)

$f(x)x^3 + y = 2 \rightarrow x^3 + x = 2 \Rightarrow x^3 + x - 2 = 0 \rightarrow x^3 + x - 2 \,\lfloor\, x - 1 \,;\ x^2 + x + 2,\ 0 \rightarrow (x-1)(x^2+x+2) = 0$

$\xrightarrow{x=1} f(2) = 1$

$x^3 + x = 10 \quad x^3 + x - 10 = 0 \quad x^3 + x - 10 \,\lfloor\, x - 2 \,;\ x^2 + 2x + 5,\ 0 \rightarrow (x-2)(x^2+2x+5) = 0$

$\xrightarrow{x=2} f(10) = 7 \rightarrow 1 + 7 = \boxed{8}$

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۷)

$f(x) = x^3 - 6x^2 + 12x = (x-2)^3 + 8 \rightarrow (\sqrt[3]{2} + 2 - 2)^3 + 8 \rightarrow 2 + 8 = 10$

$g(x) = x^3 + 9x^2 + 27x = (x+3)^3 - 27 \rightarrow (\sqrt[3]{7} - 3 + 3)^3 - 27 \rightarrow 7 - 27 = -20$

$\frac{g(\sqrt[3]{7} - 3)}{f(\sqrt[3]{2} + 2)} = \frac{-20}{10} = \boxed{-2}$

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۸)

$z = \sqrt{x - 4} \rightarrow x - 4 \geq 0 \rightarrow x \geq 4 \rightarrow z \geq 0 \rightarrow x = z^2 + 4$

$f(x) = \sqrt{z^2 + 4z + 4} \rightarrow \sqrt{(z+2)^2} \rightarrow z + 2$

$g(x) = \sqrt{z^2 - 4z + 4} \rightarrow \sqrt{(z-2)^2} \rightarrow |z - 2|$

$f(x) + g(x) = 2z = 2\sqrt{x - 4} \rightarrow a + b\sqrt{x + c} \rightarrow a = 0 \quad b = 2 \quad c = -4$

$\frac{a+b}{c} = \frac{2}{-4} = \boxed{-\frac{1}{2}}$

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۹)

$f(x) = \frac{x+2}{(x-1)(x-3)} \rightarrow R - \{1, 3\}, \quad \frac{g}{f} = \{(2, -\frac{1}{4})(0, 9)\}$

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۱۰)

الف) $f(2x) = \{(\frac{1}{2}, 2)(\frac{3}{2}, -1)(2, 2)(-\frac{1}{2}, 6)\}$

ب) $f(x^2) = \{(\pm 1, 2)(\pm\sqrt{3}, -1)(\pm 2, 2)\}$

ج) $2g^2(x) + 1 = \{(-2, 19)(1, 3)(3, 9)(-1, 1)\}$

د) $\frac{2f}{g} = \{(1, -4)(3, -1)\}$