حل تمرین

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$f(x)=(\frac{1}{2})^{x-2}-1 \to (\frac{1}{2})^{...}$ $2^{2-x}-1$

$x^2 f(x+1)\le 0 \Rightarrow x^2(2^{1-x}-1)$ $\quad 2^{1-x}-1\le 0 \to x=1$

$2^{1-x}=1 \to 1-x=0$

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$\begin{array}{ccccc} - & 0 & + & 1 & - \end{array}$ $\quad D_f=[0,+1]$

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$f(x)=\sqrt{x+|x-2|}$ $\quad D_{f(-x)}=?$

$f(-x)=\sqrt{-x+|2-x|}\ge 0$

$x-|x-2|\le 0 \quad x\le |x-2|$ به توان ۲

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$x^2\le (x-2)^2 \to x^2\le x^2-4x+4 \to x\le 1$

$D_{f(-x)}=(-\infty,1]$

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$g=\sqrt{\frac{x-1}{f(x)}}$

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$x-1=0 \to x=1$ $\quad \begin{array}{ccc} - & 1 & + \end{array}$

$f(x)=$ $\begin{array}{ccccccccc} + & -5 & + & -2 & - & 2 & + & 3 & - \end{array}$ $\quad D_g=(-4,1]\cup(2,3)$

$\frac{x-1}{f(x)} \to$ $\begin{array}{ccccccccccc} - & -5 & - & -2 & + & 1 & - & 2 & + & 3 & - \end{array}$

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$y_1=\frac{1}{9x^4-7x^2-4x-2}$ $\quad y_2=\frac{1}{3x^2+ax+b}$

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$3x^2(3x^2-\frac{7}{3}-\frac{4}{3x}-\frac{1}{x^2})$

$3x^2-x-3=3x^2+ax+b$

$a=-1,\ b=-3$

$-\frac{7}{3}-\frac{1}{x^2}=b$

$-\frac{4}{3x}=ax \to a=-\frac{4}{3}$

حل سوالات

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$f(x) = x^3 + x$

?

$$y = \sqrt{f(x) - f\left(\frac{4}{x}\right)} \geq 0 \rightarrow f(x) \geq f\left(\frac{4}{x}\right)$$

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$$x^3 + x \geq \left(\frac{4}{x}\right)^3 + \frac{4}{x} \rightarrow x^3 + x - \frac{4}{x} = \frac{x^4 + x^2 - 4}{x} \geq \frac{4^3}{x^3}$$

$$x^2(x^4 + x^2 - 4) \geq 4^3$$

۶-

$f(x^3 + x) = 2x^2 - 1$ $\qquad f(10) + f(2) = 8$

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$x = 2 \quad f(8 + 2) = 7$ $\qquad f(10) = 7 \qquad f(2) = 1$

$x = 1 \longrightarrow f(2) = 1$

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$$f(x) = \underbrace{x^3 - 6x^2 + 12x - 8}_{(x-2)^3} + 8 = (x-2)^3 + 8$$

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$$g(x) = \underbrace{x^3 + 9x^2 + 27x + 27}_{(x+3)^3} - 27 = (x+3)^3 - 27$$

$$\frac{g(\sqrt[3]{7} - 3)}{f(\sqrt[3]{2} + 2)} = \frac{(\sqrt[3]{7} - 3 + 3)^3 - 27}{(\sqrt[3]{2} + 2 - 2)^3 + 8} = \frac{-20}{10} = -2$$

۸-

$f(x) = \sqrt{x + 4\sqrt{x - 4}}$ $\qquad g(x) = \sqrt{x - 4\sqrt{x - 4}}$ $\qquad x \geq 4$

$$f(x) + g(x) = a + b\sqrt{x + c}$$

$$(f(x) + g(x))^2 = f^2_{(x)} + g^2_{(x)} + 2f(x)g(x) = x + 4\sqrt{x-4} + x - 4\sqrt{x-4}$$

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$$2x + 2\sqrt{(x + 4\sqrt{x-4})(x - 4\sqrt{x-4})} = 2x + 2|x - 8| =$$

$$x^2 - 16(x - 4) = x^2 - 16x + 64$$

$$(x - 8)^2$$

حل تمرین

$8 \le x \quad \rightarrow (f(x)+g(x))^2 = 4x-16 \rightarrow f(x)+g(x) = \sqrt{4(x-4)}$

$2\sqrt{x-4} \quad \rightarrow a+b=2 \qquad c=-4 \quad \rightarrow \frac{a+b}{c} = \frac{2}{-4} = -\frac{1}{2}$

۲۰ ب:

$x+4\sqrt{x-4} -4+4 = x-4+4\underbrace{\sqrt{x-4}}_{t}+4 \rightarrow t^2+4t+4$

$(t+2)^2 = (t+2) = \sqrt{x-4}+2$ } روش دوم: $\sqrt{(t-2)^2} = t-2 = \sqrt{x-4}-2 \rightarrow f(x)+g(x) = 2\sqrt{x-4}-4$

$f(x) = \frac{x+2}{x^2-4x+3}$ $\qquad g(x) = \{(2,1),(1,0),(3,5),(0,4)\}$

$(x-3)(x-1)$

$D_f = \mathbb{R} - \{3,1\}$ حاصل $\frac{g}{f} = \{(0,\frac{4}{\frac{2}{3}}),(2,\frac{1}{-4})\}$ (ج)

$x \neq -2 \rightarrow \frac{g}{0}$ تعریف نشده $\qquad \frac{g}{f} = \{(0,6),(2,-\frac{1}{4})\}$

۱۵ -

الف $g(x) = \{(-2,2),(1,-1),(3,2),(-1,0)\}$

$f(x) = \{(4,2),(1,2),(3,-1),(-1,6)\}$ (۱)

$f(2x) = \{(2,2),(\cancel{\frac{1}{2}},2),(\frac{3}{2},-1),(-\frac{1}{2},6)\}$

$f(x^2) = \{(2,2),(1,2),(3,-1)\}$

$2g^2(x)+1 = \{(-2,9),(1,3),(3,9),(-1,1)\}$

$\frac{2f}{g} = \{(1,-1),(3,-4),(-1,\cdot)\}$ $\quad \{(1,-4)(3,-1)\}$

۱۵ ب) $\begin{cases} x^2=1 \rightarrow x=\pm 1 \\ x^2=3 \rightarrow x=\pm\sqrt{3} \\ x^2=4 \rightarrow x=\pm 2 \\ x^2=-1 \rightarrow \times \end{cases} \rightarrow \{(\pm 1,2)(\pm\sqrt{3},-1)(\pm 2,2)\}$

۵)

$$\frac{(x^{۲}-۴)(x^{۴}+۵x^{۲}+۱۴)}{x^{۲}} \geq ۰ \qquad D_f=[-۲,۰)\cup[۲,+\infty)$$

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$-$		$+$		$-$		$+$