Subject:

سارینا اسپنانی - یازدهم دختر C - شماره تکلیف ۵

① $f(x) = \left(\frac{1}{2}\right)^{x-2} - 1 \rightarrow f(x+1) = \left(\frac{1}{2}\right)^{x-1} - 1$

$\underbrace{x^3}_{0} x \underbrace{(2^{1-x} - 1)}_{0} \geq 0$

$\rightarrow 2^{1-x} - 1 = 0 \rightarrow 2^{1-x} = 1 \xrightarrow{2^0} x = 1$ $\Big\}$ $\boxed{D_f = R - \{0, 1\}}$

② $f(x) = \sqrt{x + |x+2|} \rightarrow f(-x) = \sqrt{|2-x| - x} \rightarrow |2-x| - x \geq 0 \rightarrow$

$2 - x \geq x \rightarrow 1 \geq x$

$2 - x \leq -x \rightarrow 2 \leq 0 \rightarrow$ ممکن نیست $\Big\} \rightarrow \boxed{D_f = (-\infty, 1]}$

③ $y = \sqrt{\frac{x-1}{f(x)}} \rightarrow$ $x - 1 \rightarrow 1$, $f(x) \rightarrow -4, 2, 3$

-4		1		2		3	
$-$	$+$		$-$		$+$		$-$

$\frac{x-1}{f(x)} \geq 0 \rightarrow \boxed{D_f = (-4, 1] \cup (2, 3)}$

④ $D_{y_1} = D_{y_2} = R - (z) \rightarrow$ ریشه‌های ۲ معادله برابر است. چون در مخرج قرار دارند و تنها شرط نبودن در دامنه در این تابع صفر شدن مخرج است.

$9x^4 - 6x^2 - x^2 - 4x - 4 + 1$

$\underbrace{(9x^4 - 6x^2 + 1)}_{(3x^2-1)^2} - \underbrace{x^2 - 4x - 4}_{(x+2)^2}$

$(3x^2 - 1)^2 - (x+2)^2$

$(3x^2-1)^2 - (x+2)^2 = 0 \rightarrow 3x^2 - 1 = x + 2 \rightarrow 3x^2 - x = 3 \rightarrow$

$3x^2 - x - 3 = 0 \rightarrow 3x^2 - x - 3 = 3x^2 + ax + b \rightarrow a = -1, \; b = -3 \rightarrow$

$\boxed{\sqrt{-(a+b)} = \sqrt{4} = 2}$

Subject:

⑤

$f(x) = x^3 + x \qquad y = \sqrt{f(x) - f(\frac{4}{x})} \to f(x) - f(\frac{4}{x}) \geq 0 \to$

$f(x) \geq f(\frac{4}{x}) \to x^3 + x \geq \frac{64}{x^3} + \frac{4}{x} \to x^3 - \frac{64}{x^3} \geq x - \frac{4}{x} \to$

$(x - \frac{4}{x})(x^2 + \frac{16}{x^2} + 4) + x - \frac{4}{x} \geq 0 \to x^2 + \frac{16}{x^2} + 4 + 1 \geq 0 \to$

$(x^4 + 16 + 5x^2)(x - \frac{4}{x}) \geq 0 \qquad \Delta > 0 \to +$ همیشه

$\neq 0 \quad \swarrow x^2$

$x - \frac{4}{x} \to 2, -2 \to$

	-2		0		2	
	0	$+$	$\nexists$	$-$	0	$+$

$\to \boxed{D_f = (-\infty, -2] \cup [2, +\infty)}$

⑥

$f(x^3 + x) = 2x^2 - 1 \to f(2) + f(10) =$

$x^3 + x = 2 \to x = 1 \to 2 \times 1 - 1 = 1$

$x^3 + x = 10 \to x = 2 \to 2 \times 4 - 1 = 7$

$\boxed{f(2) + f(10) = 1 + 7 = 8}$

⑦

$f(x) = x^3 - 6x^2 + 12x$

$f(\sqrt[3]{2} + 2) = (2 + 8 + \cancel{6\sqrt[3]{4}} + \cancel{12\sqrt[3]{2}}) - 6(\cancel{\sqrt[3]{4}} + \cancel{4\sqrt[3]{2}} + \cancel{4})$

$+ 12(\cancel{\sqrt[3]{2}} + \cancel{2}) = 10$

$g(x) = x^3 + 9x^2 + 27x$

$g(\sqrt[3]{7} - 3) = (7 - 27 - 9\sqrt[3]{49} + 27\sqrt[3]{7}) + 9(\sqrt[3]{49} - 6\sqrt[3]{7} + 9)$

$+ 27(\sqrt[3]{7} - 3) = 7 - 27 = -20$

$\to \frac{g(\sqrt[3]{7} - 3)}{f(\sqrt[3]{2} + 2)} = \boxed{\frac{-20}{10} = -2}$

Subject:

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$$f(x)=\sqrt{(x-4)+4\sqrt{x-4}+4} \rightsquigarrow$$

$$f(x)=\sqrt{\left(\sqrt{x-4}+2\right)^2} \rightsquigarrow f(x)=\left|\sqrt{x-4}+2\right|$$

$$g(x)=\sqrt{(x-4)-4\sqrt{x-4}+4} \rightsquigarrow g(x)=\left|\sqrt{x-4}-2\right|$$

$$f(x)+g(x)=a+b\sqrt{x+c} \xrightarrow{4\leq x<8} \sqrt{x-4}+2+2-\sqrt{x-4}=4$$

$$x\geq 8 \to \sqrt{x-4}+2+\sqrt{x-4}-2=0+2\sqrt{x-4}$$

$$0+2\sqrt{x-4}=\underset{0}{a}+\underset{2}{b}\sqrt{x+\underset{-4}{c}} \to \boxed{\frac{a+b}{c}=\frac{0+2}{-4}=\frac{-1}{2}}$$

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$$x^2-4x+3\neq 0 \to (x-1)(x-3)\neq 0$$

$$1 \qquad 3$$

$$\frac{x+2}{x^2-4x+3}\neq 0 \qquad x+2 \to -2 \qquad x^2-4x+3 \to 1,3$$

$$f(2)=-4 \qquad f(0)=\frac{2}{3}$$

$$\boxed{g_f=\left\{\left(2,-\frac{1}{4}\right),(0,9)\right\}}$$

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$$f(2x)=\left\{\left(\frac{1}{2},2\right),\left(\frac{3}{2},-1\right),(2,2),\left(-\frac{1}{2},9\right)\right\}$$

$$f(x^2)=\left\{(1,2),(\sqrt{3},-1),(2,2)\right\}$$

$$2g^2(x)+1=\left\{(-2,19),(1,3),(3,9),(-1,1)\right\}$$

$$\frac{2f}{g}=\left\{(1,-4),(3,-1)\right\}$$