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۵ ①

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$

$$\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \Rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0$$

$$\frac{-2x+1}{x(x-1)} \geq 0$$

$x = \frac{1}{2}$
 $x=0 \leftarrow$ $x=1 \rightarrow$

$$\begin{array}{c} 0 \quad \frac{1}{2} \quad 1 \\ + \quad - \quad + \quad - \end{array}$$

$$D_f = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} = \frac{\frac{x-2x-1}{x(x+1)}}{\frac{3x+6+x-1}{(x-1)(x+2)}} = \frac{\frac{-(x+1)}{x(x+1)}}{\frac{4x+5}{(x-1)(x+2)}}$

مخرج در صورت نباید صفر شود $\Rightarrow x \neq 0, x \neq -1$

$$D_f = \mathbb{R} - \left\{-1, -\frac{5}{4}, -1, 0, 1\right\}$$

مخرج نباید صفر شود $\Rightarrow x \neq 1, x \neq -2, x \neq -\frac{5}{4}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)(32 - x^2)}$

$$\left(\left(\frac{1}{x}\right)^2 - 9\right) - (32 - x^2) \geq 0$$

$$\left(\frac{1}{x}\right)^2 - 9 = 0 \rightarrow x = -\frac{1}{3}$$

$$\begin{array}{c} -\frac{1}{3} \quad \frac{5}{2} \\ + \quad - \quad + \end{array}$$

$$32 - x^2 = 0 \rightarrow x = \pm \frac{5}{2}$$

$$D_f = (-\infty, -\frac{1}{3}] \cup \left[\frac{5}{2}, +\infty\right)$$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3 \rightarrow \sqrt{y+1} = 3 - \sqrt[4]{x-1}$

بهرای $y \geq -1 \rightarrow y+1 = 9 - 6\sqrt[4]{x-1} + \sqrt{x-1} \rightarrow y = 8 - 6\sqrt[4]{x-1} + \sqrt{x-1}$

$$x-1 \geq 0 \rightarrow x \geq 1 \quad \text{①}$$

$$3 - \sqrt[4]{x-1} \geq 0 \rightarrow \sqrt[4]{x-1} \leq 3 \xrightarrow{\text{بهرای}} x-1 \leq 81 \rightarrow x \leq 82 \quad \text{②}$$

$$D_f = \text{①} \cap \text{②} = [1, 82]$$

$$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

$$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \quad \frac{-1}{+} \frac{2}{-} \frac{+}{+} \rightarrow (-\infty, -1) \cup (2, +\infty) \quad \textcircled{I}$$

$$x^2 - 1 \geq 0 \rightarrow (x-1)(x+1) \geq 0 \quad \frac{-1}{+} \frac{1}{-} \frac{+}{+} \rightarrow (-\infty, -1] \cup [1, +\infty) \quad \textcircled{II} \star$$

عبارت زیر را دیکال به ازای x های که عبارت در آن ها تعریف شود یعنی $(-\infty, -1] \cup [1, +\infty)$ جواب دیکال + می شود همواره. $\textcircled{III}$

$$D_f = \textcircled{I} \cap \textcircled{II} \cap \textcircled{III} = (-\infty, -1) \cup (2, +\infty)$$

$$\sqrt{3+ax-x^2} \rightarrow 3+ax-x^2 \geq 0 \rightarrow x^2-ax-3 \leq 0$$

$$-1 \times b = 3 \rightarrow b = \frac{3}{-1} \rightarrow a = -2 + \frac{3}{-1} = \frac{-5}{1}$$

$$\rightarrow a+b = \frac{3}{-1} - \frac{1}{-1} = \textcircled{1}$$

$$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 3 & x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x) - x} \rightarrow f(x) - x \geq 0 \rightarrow f(x) \geq x$$

$$\left. \begin{array}{l} \text{if } x \geq 1 \rightarrow x^2 - 2 \geq x \rightarrow x \geq 1 \quad \textcircled{I} \\ \text{if } x < 1 \rightarrow x^2 + 3 \geq x \rightarrow x \geq -2 \quad \textcircled{II} \end{array} \right\} D_f = \textcircled{I} \cup \textcircled{II} = [-2, +\infty)$$

$$f(x) = \begin{cases} (a+1)(x+1) & x > 1 \\ 3a+1x & x \leq 1 \end{cases}$$

$$1f(0) = f(-1) + a \rightarrow 1x(1)(a+1) = 3a + 1x(-1) + a \Rightarrow 1a + 1x = 3a - 1$$

$$\rightarrow 1a = -1 \rightarrow \boxed{a = \frac{-1}{1}}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + r$$

$$\frac{1}{r+\sqrt{c}} = r - \sqrt{c}$$

$$f(r+\sqrt{c}) + f(r-\sqrt{c})$$

$$f(r) + f\left(\frac{1}{r}\right) = \sqrt{r} + \frac{1}{\sqrt{r}} + r + \sqrt{\frac{1}{r}} + \frac{1}{\sqrt{\frac{1}{r}}} + r$$

$$= r\left(\sqrt{r} + \frac{1}{\sqrt{r}}\right) + \varepsilon$$

$$\rightarrow r\left(\sqrt{r+\sqrt{c}} + \frac{1}{\sqrt{r+\sqrt{c}}}\right) + \varepsilon = r\left(\sqrt{r+\sqrt{c}} + \sqrt{r-\sqrt{c}}\right) + \varepsilon$$

$$A' = (r - \sqrt{c}) + (r + \sqrt{c}) + r\sqrt{(r+\sqrt{c})(r-\sqrt{c})} = 4 \rightarrow A = \sqrt{4}$$

$$f(r+\sqrt{c}) + f(r-\sqrt{c}) = rA + \varepsilon = \boxed{r\sqrt{4} + \varepsilon}$$

$$r f(x) - r f(-x) = r x^r - x$$

$$r f(-x) - r f(x) = r x^r + x$$

$$rx(r f(x) - r f(-x)) = r x^r - x$$

$$rx(-r f(x) + r f(-x)) = r x^r + x$$

$$-2f(x) = r \cdot x^r + x$$

$$f(x) = \frac{r \cdot x^r + x}{-2} = -\frac{r x^r}{2} - \frac{x}{2}$$

$$= \frac{-2(r \cdot x + 1)}{2}$$

$$(x+r)f(x) - rx f(x+r) = rx^r - mx + rm - 1$$

$$x=0 \rightarrow (r f(0) = rm - 1) \times -c$$

$$x=-r, 9 f(0) = 19 + rm + rm - 1$$

$$0 = 9m + 5 + 19 + 2m - 1 \Rightarrow rm = 11 \rightarrow m = \frac{9}{r}$$

$$r f(0) = r\left(\frac{9}{r}\right) - 1 = \frac{16 - r}{r} = \frac{r-16}{r} \rightarrow f(0) = \frac{r-16}{r} = \frac{r-16}{r} = \boxed{9/16}$$

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = a\left(x + \frac{1}{x}\right) + 2b = rx - 1r + \frac{c}{x}$$

$$\frac{rx^r - 1rx + c}{x} = rx - 1r + \frac{c}{x}$$

$$a = r$$

$$b = -r$$

$$f(-1) = \boxed{-9}$$