

الف) $f(u) = \sqrt{\frac{u-1}{u}} - \frac{u}{u-1} \geq 0 \Rightarrow \frac{(u-1)^2 - u^2}{u(u-1)} \geq 0 \Rightarrow \frac{u^2 - 2u + 1 - u^2}{u(u-1)} \geq 0 \Rightarrow \frac{-2u+1}{u(u-1)} \geq 0$

$\Rightarrow \frac{0}{+} \frac{1}{-} \frac{1}{+} \Rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, 1) \Rightarrow$ جواب

ب) $f(u) = \frac{1}{u+1} - \frac{2}{u} \geq 0 \Rightarrow \frac{1}{u+1} \geq \frac{2}{u} \Rightarrow u \neq -1, \frac{2}{u} \rightarrow u \neq 0 \rightarrow \mathbb{R} - \{0, -1\}$
 $\frac{u}{u-1} + \frac{1}{u+2} \geq 0 \Rightarrow \frac{u(u+2) + (u-1)}{(u-1)(u+2)} \geq 0 \Rightarrow \frac{u^2 + 3u - 1}{(u-1)(u+2)} \geq 0$
 $\Rightarrow \frac{3}{+} \frac{1}{-} \frac{1}{+} \Rightarrow D_f = \mathbb{R} - \{-2, -\frac{1}{3}\}$

الف) $f(u) = \sqrt{(\frac{1}{u})^4 - 9} [32 - \varepsilon^u] \geq 0 \Rightarrow (\frac{1}{u})^4 - 9 \leq 0 \Rightarrow (\frac{1}{u})^4 = 9 \Rightarrow u = -\frac{2}{3}$
 $\Rightarrow \frac{-2}{+} \frac{9}{-} \frac{1}{+} \Rightarrow D_f = (-\infty, -\frac{2}{3}] \cup [\frac{2}{3}, +\infty) \Rightarrow$ جواب

ب) $\sqrt{u-1} + \sqrt{u+1} = 3 \Rightarrow \sqrt{u+1} = 3 - \sqrt{u-1}$
 $\Rightarrow \sqrt{u-1} \rightarrow u-1 \geq 0 \rightarrow u \geq 1$
 $3 - \sqrt{u-1} \geq 0 \Rightarrow \sqrt{u-1} \leq 3 \Rightarrow u-1 \leq 9 \Rightarrow u \leq 10$
 $\Rightarrow D_f = [1, 10]$

$f(u) = \frac{\log_e(u^2 - u - 2)}{\sqrt{u^2 - 1} + 1} \geq 0 \Rightarrow \sqrt{u^2 - 1} + 1 \neq 0 \Rightarrow \sqrt{u^2 - 1} \neq -1 \checkmark$
 $\Rightarrow \sqrt{u^2 - 1} \rightarrow u^2 - 1 \geq 0 \rightarrow u^2 \geq 1 \rightarrow u \geq 1 \text{ or } u \leq -1 \Rightarrow D_f = (-\infty, -1] \cup [1, +\infty)$
 $\star \rightarrow \log_e(u^2 - u - 2) \geq 0 \Rightarrow u^2 - u - 2 \geq 1 \Rightarrow u^2 - u - 3 \geq 0 \Rightarrow (u-2)(u+1) \geq 0 \Rightarrow u \geq 2 \text{ or } u \leq -1$
 $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

$\sqrt{3+au-u^2} \rightarrow D_f = [-2, b] \Rightarrow$
 $\star u=b \rightarrow 3+bx-\frac{1}{x}-b^2=0 \Rightarrow -b^2-\frac{1}{x}b+3=0 \Rightarrow u = \frac{-b \pm \sqrt{\Delta}}{2a} \Rightarrow \frac{1}{x} \pm \frac{\sqrt{\Delta}}{2} \Rightarrow \frac{1}{x} \pm \frac{1}{2} \Rightarrow u = -2 \text{ or } u = \frac{3}{2}$
 $\Delta = b^2 - 4ac \Rightarrow \Delta = \frac{1}{4} - (2x-1)(3) \Rightarrow \Delta = \frac{1}{4}$
 $a+b \rightarrow \frac{3}{2} - \frac{1}{2} = 1 \Rightarrow$ جواب

$g(u) = \sqrt{f(u)} - u \rightarrow f(u) - u \geq 0 \rightarrow f(u) \geq u$
 $f(u) = \begin{cases} 2u-2; & u \geq 1 \rightarrow 2u-2 \geq u \rightarrow 2u \geq 2 \rightarrow u \geq 1 \checkmark \rightarrow \text{محدوده جفتداری} \text{ I} \\ 2u+3; & u < 1 \rightarrow 2u+3 \geq u \rightarrow u \geq -3 \text{ II} \end{cases}$
 $\Rightarrow D_f = [-3, +\infty) \Rightarrow$ جواب

$$f(u) = \begin{cases} (a+1)(u+r) & ; u \geq 1 \\ ra + ru & ; u \leq 1 \end{cases}$$

$$rf(a) = f(-r) + a \xrightarrow{(*)} 12a + 12 = 3a - 2 + a \rightarrow 12a + 12 = 4a - 2 \rightarrow 10a = -14$$

$$f(a) \rightarrow (a+1)(\frac{a+r}{r}) \rightarrow va + v \rightarrow rf(a) = 12a + 12 \quad (*)$$

$$a = -\frac{9}{5}$$

$$f(-r) \rightarrow ra + rx - r \rightarrow 3a - 2 \quad (*)$$

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$$rf(u) - rf(-u) = \varepsilon u^2 - u$$

$$\times \{ rf(u) - rf(-u) = \varepsilon u^2 - u$$

$$\times \{ rf(-u) - rf(u) = \varepsilon u^2 + u$$

$$\varepsilon f(u) - 9f(u) = 14u^2 - ru + 12u^2 + 4u$$

$$- 8f(u) = 10u^2 + u$$

$$f(u) = -\frac{10u^2 + u}{8} \xrightarrow{\text{مشتق}} f'(u) = -\varepsilon u^2 + \frac{u}{8} \rightarrow \text{جواب}$$

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$$f(r+\sqrt{r}) \xrightarrow{\text{مشتق}} \frac{1}{r+\sqrt{r}} \xrightarrow{\text{بسط}} \frac{1}{r+\sqrt{r}} \times \frac{r-\sqrt{r}}{r-\sqrt{r}} = r-\sqrt{r} \rightarrow f(r+\sqrt{r}) = f(r-\sqrt{r}) \quad (*)$$

$$f(r+\sqrt{r}) + f(r-\sqrt{r}) \rightarrow f(r+\sqrt{r}) + f\left(\frac{1}{r+\sqrt{r}}\right) \rightarrow f(t) + f\left(\frac{1}{t}\right) \rightarrow$$

$$r\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right) + 2 \rightarrow r\left(\sqrt{r+\sqrt{r}} + \frac{1}{\sqrt{r+\sqrt{r}}}\right) \rightarrow r\left(\sqrt{r+\sqrt{r}} + \sqrt{r-\sqrt{r}}\right) + 2$$

$$s^2 = (r-\sqrt{r})(r+\sqrt{r}) + r\sqrt{(r-\sqrt{r})(r+\sqrt{r})} \rightarrow s^2 = r \rightarrow s = \sqrt{r} \quad \times$$

$$f(r+\sqrt{r}) + f(r-\sqrt{r}) = rs + 2 \rightarrow f(r+\sqrt{r}) + f(r-\sqrt{r}) = r \times \sqrt{r} + 2$$

$$= r\sqrt{r} + 2 \rightarrow \text{جواب}$$

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$$(u+r)f(u) - ru f(u+r) = \varepsilon u^2 - mu + rm - 1$$

$$\hookrightarrow u=0 \rightarrow rf(0) = rm - 1$$

$$\hookrightarrow u=-r \rightarrow -rf(0) = 14 + \frac{rm+rm-1}{rm}$$

$$\rightarrow \begin{cases} rf(0) = rm - 1 \\ -rf(0) = 14 + \frac{rm+rm-1}{rm} \end{cases}$$

$$9m - 14 = 14 + \frac{2rm-1}{rm}$$

$$\varepsilon m = 14 \rightarrow m = \frac{9}{r} \quad (*)$$

$$\rightarrow rf(0) = rm - 1 \xrightarrow{(*)} rf(0) = r \times \frac{9}{r} - 1 \rightarrow rf(0) = \frac{rv}{r} - 1 \rightarrow rf(0) = \frac{va}{r}$$

$$\rightarrow f(0) = \frac{va}{\varepsilon} \rightarrow \text{جواب}$$

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$$f(u) + f\left(\frac{1}{u}\right) = \frac{ru^2 - 12u + r}{u} \xrightarrow{u=-1} f(-1) + f(-1) = \frac{r(-1)^2 - 12(-1) + r}{-1}$$

$$\rightarrow rf(-1) = \frac{r+12+r}{-1} \rightarrow rf(-1) = -14 \rightarrow f(-1) = -9$$

جواب

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