۱- الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \Rightarrow \frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{(x-1)^2-x^2}{x(x-1)}\geq 0 \to \frac{-2x+1}{x(x-1)}\geq 0$

$\to$ $\begin{array}{c|c|c|c} & 0 & \frac{1}{2} & 1 \\ + & \nexists\ - & + & \nexists\ - \end{array}$ $\Rightarrow D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \to D_f=\mathbb{R}-\{-2,-1,0,1\}$

۲- الف) $f(x)=\sqrt{\left((\frac{1}{2})^x-9\right)(32-4^x)} \Rightarrow \left((\frac{1}{2})^x-9\right)(32-4^x)\geq 0 \Rightarrow \begin{array}{ccc} -2 & & \frac{5}{2} \\ - & + & - \end{array} \Rightarrow D_f=[-2,\frac{5}{2}]$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \to \sqrt{y+1}=3-\sqrt[4]{x-1} \to y+1=9-6\sqrt[4]{x-1}+\sqrt{x-1} \to y=(3-\sqrt[4]{x-1})^2-1$

(1) $3-\sqrt[4]{x-1}\geq 0 \to$; $\Rightarrow x\geq 1$

(1) $\to \sqrt[4]{x-1}\leq 3 \Rightarrow x-1\leq 81 \Rightarrow x\leq 82 \xrightarrow{\cap} 1\leq x\leq 82 \Rightarrow D_f=[1,82]$

۳- $f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$ $\to \begin{cases} x^2-x-2>0 \to \begin{array}{ccc} -1 & & 2 \\ + & - & + \end{array} \\ \sqrt{x^2-1}+1\neq 0 \to \sqrt{x^2-1}\neq -1 \to \text{همواره برقرار} \\ x^2-1\geq 0 \to x^2\geq 1 \Rightarrow x\geq 1,\ x\leq -1 \end{cases}$ $\Rightarrow (-\infty,-1)\cup(2,+\infty)$

۴- $f(x)=\sqrt{3+ax-x^2}$ $\quad D_f=[-2,b] \Rightarrow x^2-ax-3\leq 0$

$\begin{array}{ccc} -2 & & b \\ - & + & - \end{array}$ $\Rightarrow (x+2)(x-b)\leq 0 \Rightarrow (x-2)(x-\frac{3}{2})\leq 0 \Rightarrow b=\frac{3}{2}$

$\to a=b+(-2) \Rightarrow a+b=2b-2 \Rightarrow \boxed{a+b=3-2=1}$

۵- $f(x)=\begin{cases} 3x-2 & ; x\geq 1 \\ 2x+3 & ; x<1 \end{cases}$ $\quad g(x)=\sqrt{f(x)-x} \Rightarrow f(x)-x\geq 0$

$f(x)=\{(1,1)(3,3),\dots\} \Rightarrow \begin{array}{ccc} 1 & & 3 \\ + & - & + \end{array} \Rightarrow D_g=(-\infty,1]\cup[3,+\infty)$

۶- $f(x)=\begin{cases} (a+1)(x+2) & ; x>1 \\ 2a+2x & ; x\leq 1 \end{cases}$ $\quad 2f(5)=f(-2)+a$

$2(a+1)\times 7=2a-4+a \Rightarrow 14a+14=4a-4 \Rightarrow \boxed{a=-\frac{9}{5}}$

۷- $f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$

$(2-\sqrt{3})(2+\sqrt{3})=1 \Rightarrow$ این دو عکس هم اند

$\overbrace{f(\underbrace{2-\sqrt{3}}_{a})+f(2+\sqrt{3})}^{A}=\sqrt{a}+\frac{1}{\sqrt{a}}+2+\frac{1}{\sqrt{a}}+\sqrt{a}+2=2(\sqrt{a}+\frac{1}{\sqrt{a}}+2)=2(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}+2)=A$

توان ۲ $\quad A\to 2\left(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}\right)+4=A \to 4(2-\sqrt{3}+2+\sqrt{3}-2(1))=(A-4)^2 \Rightarrow 24=(A-4)^2$

$\to A-4=\sqrt{24} \Rightarrow A=2\sqrt{6}+4 \Rightarrow \boxed{f(2-\sqrt{3})+f(2+\sqrt{3})=2\sqrt{6}+4}$

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$$8-\ (2f(x)-3f(-x)=4x^2-x)\times 2 \rightarrow 4f(x)-6f(-x)=8x^2-2x$$
$$(2f(-x)-3f(x)=4x^2+x)\times 3 \Rightarrow 6f(-x)-9f(x)=12x^2+3x$$
$$\Rightarrow \boxed{f(x)=-4x^2-\frac{1}{5}x}$$

$$9-\ (x+2)f(m)-3xf(x+2)=4x^2-mx+3m-1$$

$$\underline{x=-2},\ 6f(0)=5m-15$$
$$\underline{x=0},\ 2f(0)=3m-1$$
$$\Rightarrow 5m+15=9m-3 \Rightarrow 4m=18 \Rightarrow m=\frac{9}{2} \Rightarrow \boxed{f(0)=\frac{25}{4}}$$

$$10-\ f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x}$$
$$\underline{x=-1},\ f(-1)+f(-1)=\frac{3+12+3}{-1} \Rightarrow 2f(-1)=-18 \Rightarrow \boxed{f(-1)=-9}$$