

سوال (۱)

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} = \sqrt{\frac{x^2 - 2x + 1 - x^2}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}}$

$$\Rightarrow D_f: (-\infty, 0) \cup [\frac{1}{2}, 1)$$

ب) $f(x) = \frac{1}{x+1} - \frac{2}{x}$

$$\Rightarrow D_f: \mathbb{R} - \left\{ -\frac{5}{2}, -1, -2, 0, 1 \right\}$$

سوال (۲)

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^x - 9\right)(3x - 2x)}$

$$\Rightarrow D_f: (-\infty, -2] \cup [2, \infty)$$

ب) $\sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = -\sqrt{x-1} + 3 \Rightarrow 3 - \sqrt{x-1} \geq 0 \Rightarrow 1 \geq x-1$

$$\Rightarrow x \leq 1 \Rightarrow (-\infty, 1] \quad \cup \quad x-1 \geq 0 \Rightarrow x \geq 1 \Rightarrow [1, \infty) \Rightarrow D_f: [1, 12]$$

سوال (۳)

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

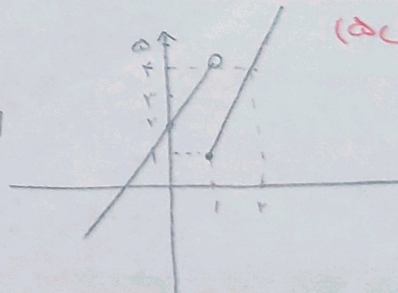
$$\Rightarrow D_f: (-\infty, -1) \cup (2, \infty)$$

سوال (۴)

$\sqrt{3+ax-x^2}$ $D_f: [-2, b]$ $a+b=?$

$$a+b = -\frac{1}{2} + \frac{3}{2} = 1$$

الف) $f(x) = \begin{cases} 3x-2 & x \geq 1 \\ 2x+3 & x < 1 \end{cases}$ $D_g: ?$ $g(x) = \sqrt{f(x)-x}$

$$\Rightarrow D_g: [-3, \infty)$$


سوال (۵)

$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 2a+2x & x \leq 1 \end{cases}$, $2f(5) = f(-2) + a$ $a=?$

$$\Rightarrow 10a = -18 \Rightarrow a = -1.8$$

سوال (۷)

$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$ $f(2-\sqrt{3}) + f(2+\sqrt{3}) = ?$

$$\frac{1}{\sqrt{2-\sqrt{3}}} = \frac{1}{\sqrt{2-\sqrt{3}}}$$

$$2(1 + \sqrt{2-\sqrt{3}})(1 + \frac{1}{\sqrt{2-\sqrt{3}}}) = 2(2 + \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}}) = 2(2 + \sqrt{6}) = 4 + 2\sqrt{6}$$

$$\frac{3-\sqrt{3}}{\sqrt{2-\sqrt{3}}} = (3-\sqrt{3})(\sqrt{2+\sqrt{3}}) \rightarrow (1-\sqrt{3})(2+\sqrt{3}) = 4\sqrt{3} \rightarrow \sqrt{3}$$

$$2f(x) - 3f(-x) = 5x^2 - x$$

$f(x) = ?$ (سوال ۸)

$$\times 2 \rightarrow 4f(x) - 6f(-x) = 10x^2 - 2x$$

$$2f(x) - 3f(-x) = 5x^2 - x \xrightarrow{x \rightarrow -x} 2f(-x) - 3f(x) = 5x^2 + x \xrightarrow{\times 2} 4f(-x) - 6f(x) = 10x^2 + 2x$$

$$\begin{cases} 4f(x) - 6f(-x) = 10x^2 - 2x \\ 4f(-x) - 6f(x) = 10x^2 + 2x \end{cases}$$

$$-2f(x) = 2x^2 + x \rightarrow f(x) = -x^2 - \frac{x}{2}$$

$$(2+x)f(x) - 3xf(x+x) = 5x^2 - mx + 3m - 1$$

$f(0) = ?$ (سوال ۹)

$$x=0 \rightarrow 2f(0) = 3m - 1$$

$$x=-2 \rightarrow 4f(0) = 14 + 2m + 3m - 1 \rightarrow 4f(0) = 13 + 5m \rightarrow f(0) = \frac{13 + 5m}{4}$$

$$\Rightarrow m = \frac{4}{5}$$

$$2f(0) = \frac{13 + 5 \cdot \frac{4}{5}}{2} = \frac{13 + 4}{2} = \frac{17}{2}$$

$$\Rightarrow f(0) = \frac{17}{4}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{5x^2 - 12x + 3}{x}$$

$f(-1) = ?$ (سوال ۱۰)

$$2f(-1) = \frac{5(-1)^2 - 12(-1) + 3}{-1} = \frac{5 + 12 + 3}{-1} = -20 \rightarrow f(-1) = -10$$