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نام و نام خانوادگی ......فائقه رجایی...... پاسخنامه تشریحی تکلیف شماره ..۲.... کلاس ....

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الف) $\frac{(x-1)^2-x^2}{x(x-1)} \geq 0 \to \frac{x^2+1-2x-x^2}{x(x-1)} \geq 0 \to \frac{1-2x}{x(x-1)} \geq 0$

	0		$\frac{1}{2}$		1	
$+$		$-$		$+$		$-$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

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ب) $\frac{3}{x-1} + \frac{1}{x+2} = \frac{3(x+2)+x-1}{(x-1)(x+2)} = \frac{4x+5}{(x-1)(x+2)} \neq 0 \to 4x+5 \neq 0 \to x \neq -\frac{5}{4}$

$x+1 \neq 0 \to x \neq -1 \quad x-1 \neq 0 \to x \neq 1 \quad x+2 \neq 0 \to x \neq -2 \quad D_f = \mathbb{R} - \{-\frac{5}{4}, -1, -2, 0, 1\}$

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الف) $(\frac{1}{3})^x - 9 = 0 \to (\frac{1}{3})^x = 3^2 \to x = -2 \qquad 2^5 = 2^{2x} \to x = \frac{5}{2}$

	-2		$\frac{5}{2}$	
$+$		$-$		$+$

$D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

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ب) $x-1 \geq 0 \to \boxed{x \geq 1}$

$\sqrt{y+1} = 3 - \sqrt[4]{x-1} \to 3 - \sqrt[4]{x-1} \geq 0 \to \sqrt[4]{x-1} \leq 3 \to -81 \leq x-1 \leq 81 \to \boxed{x \leq 82}$

$D_f = [1, 82]$

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$x^2-x-2 = (x-2)(x+1) > 0 \to$

	-1		2	
$+$		$-$		$+$

$x < -1$ یا $x > 2$ ①

$\sqrt{x^2-1} + 1 \neq 0$ ✓

$x^2 - 1 \geq 0 \to x^2 \geq 1 \to x \geq 1$ یا $x < -1$ ②

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$D_f = (-\infty, -1) \cup (2, +\infty)$

$\downarrow$

$1 \cap 2$

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$y = \sqrt{-(x^2 - ax - 3)} \qquad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}, \quad \frac{a \pm \sqrt{a^2-4(-3)}}{2} = -2 \to \frac{a \pm \sqrt{a^2+12}}{2} = -2$

$a \pm \sqrt{a^2+12} = -4 \to \pm\sqrt{a^2+12} = -a-4 \xrightarrow{\text{به توان ۲}} \cancel{a^2} + 12 = \cancel{a^2} + 16 + 8a \to 8a = -4 \to \boxed{a = -\frac{1}{2}}$

$x = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4}+12}}{2}$

$\frac{-\frac{1}{2} - \frac{7}{2}}{2} = -2$

$\frac{-\frac{1}{2} + \frac{7}{2}}{2} = \boxed{\frac{3}{2} = b}$

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$a + b = \frac{3}{2} - \frac{1}{2} = \boxed{1}$

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$x \geq 1 \to 3x - 2 = x \to 2x = 2 \to x = 1$

$x < 1 \to 2x + 3 = x \to x = -3$

	-3		1	
$-$		$+$		$+$

$x = 2 \to 3(2) - 2 = 4 \to 4 - 2 > 0$

$x = 0 \to 2(0) + 3 = 3 \to 3 - 0 > 0$

$x = -4 \to 2(-4) + 3 = -5 \to -5 + 4 < 0$

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$D_g = [-3, +\infty)$

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فاطمه رجایی

$2(a+1)\underbrace{(5+2)}_{7} = 3a+2(-2)+a \to 14(a+1) = 4a-4 \to 14a+14 = 4a-4$ $\to 10a = -18 \to \boxed{a=-1.8}$	۶
$\underbrace{(2-\sqrt{3})(2+\sqrt{3})}_{\text{معکوس یکدیگر اند}} = 1 \to f(x)+f(\frac{1}{x}) = \sqrt{x}+\frac{1}{\sqrt{x}}+2+\frac{1}{\sqrt{x}}+\sqrt{x}+2 =$ $2(\sqrt{x}+\frac{1}{\sqrt{x}})+4 \to f(2-\sqrt{3})+f(2+\sqrt{3}) = 2(\overbrace{\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2+\sqrt{3}}}}^{A})+4$ $\frac{1}{\sqrt{2+\sqrt{3}}}\times\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2-\sqrt{3}}} = \sqrt{2-\sqrt{3}} \to (\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}})^2 = A^2$ $A^2 = 2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}+2\sqrt{4-3} \to A^2 = 6 \to A=\sqrt{6} \Rightarrow \boxed{2\sqrt{6}+4}$	۷
$2f(x)-3f(-x) = 4x^2-x \xrightarrow{\times 2} 4f(x)-\cancel{6f(-x)} = 8x^2-2x$ $2f(-x)-3f(x) = 4x^2+x \xrightarrow{\times 3} \cancel{6f(-x)}-9f(x) = 12x^2+3x$ $-5f(x) = 20x^2+x \to \boxed{f(x) = \frac{20x^2+x}{-5}}$	۸
$x=-2 \to (\underbrace{-2+2}_{0})f(-2)-3(-2)f(-2+2) = 4(-2)^2+2m+3m-1$ $\to 6f(0) = 15+5m \to 6f(0) = 15+\frac{45}{2} \to f(0) = \frac{75}{2}\times\frac{1}{6} = \boxed{\frac{75}{12}}$ $x=0 \to 2f(0)-3(0)f(0+2) = 4(0)^2-m(0)+3m-1$ $\to 2f(0) = 3m-1 \xrightarrow{\times 3} 6f(0) = 9m-3$ $9m-3 = 15+5m \to 4m = 18 \to \boxed{m=\frac{18}{4}=\frac{9}{2}}$	۹
$f(x) = ax+b \to ax+b+\frac{a}{x}+b = \frac{3x^2-12x+3}{x} \to \frac{ax^2+2bx+a}{x} = 3x^2-12x+3$ $f(\frac{1}{x}) = \frac{a}{x}+b$ $\left.\begin{matrix} a=3 \\ 2b=-12 \to b=-6 \end{matrix}\right\} \to f(x) = 3x-6 \to f(-1) = 3(-1)-6 = \boxed{-9}$	۱۰