رزا ابوصدالطالبی — یازدهم ریاضی — تکلیف ۴

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الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0$, $x \neq 0$, $x-1 \neq 0$, $x \neq 1$

$\frac{(x-1)^2 - (x)^2}{x(x-1)} \geq 0$, $\frac{x^2+1-2x-x^2}{x(x-1)} \geq 0$

$\frac{1-2x}{x(x-1)} \geq 0$, $\frac{1}{2}$; $x(x-1)$: 0, 1

	0		$\frac{1}{2}$		1	
$+$	$\circ$	$-$	$\bullet$	$+$	$\circ$	$-$
ق ق				ق ق		

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}}$, $x+1 \neq 0$, $x \neq 0$, $\neq 0$

① $x \neq 0$, $x \neq -1$

② $\frac{3(x+2)+1(x-1)}{(x+2)(x-1)} \neq 0$, $x \neq -2$, $x \neq 1$

$3x+6+x-1 = 4x+5 \neq 0$, $x \neq -\frac{5}{4}$

$D_f = \mathbb{R} - \{-2, -\frac{5}{4}, -1, 0, 1\}$

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الف) $f(x) = \sqrt{\left((\frac{1}{2})^x - 4\right)\left(32 - 4^x\right)} \geq 0$

$(2^{-x} - 2^2)(2^5 - 2^{2x}) \geq 0$; $-x = 2 \Rightarrow x = -2$, $2x = 5 \Rightarrow x = \frac{5}{2}$

	-2		$\frac{5}{2}$	
$+$	$\bullet$	$-$	$\bullet$	$+$
ق ق				ق ق

$D_f = (-\infty, -2] \cup [\frac{5}{2}, +\infty)$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3$

$\sqrt{y+1} = 3 - \sqrt[4]{x-1}$

① $x-1 \geq 0$, $x \geq 1$

$3 - \sqrt[4]{x-1} \geq 0$ ($3 \geq \sqrt[4]{x-1}$) تعریف

$81 \geq x-1$, ② $82 \geq x$

①∩② $D_f = [1, 82]$

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$f(x) = \frac{\log_2 (x^2-x-2)}{\sqrt{x^2-1} + 1}$

$(x^2-x-2) > 0$, $(x-2)(x+1) > 0$; 2 , -1

	-1		2	
$+$	$\circ$	$-$	$\circ$	$+$
ق ق				ق ق

① $(-\infty, -1) \cup (2, +\infty)$

② $\sqrt{x^2-1} + 1 \neq 0$, $\sqrt{x^2-1} \neq -1$ جواب ندارد

①∩② $\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$

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$\sqrt{3 + ax - x^2} \geq 0$, $D_f = [-2, b]$ $\Rightarrow b = \frac{3}{2}$

$a + b = -\frac{1}{2} + \frac{3}{2} = 1$ جواب

$3 + a(-2) - (-2)^2 = 0$, $3 - 2a - 4 = 0$, $-1 = 2a$, $a = -\frac{1}{2}$

$\Rightarrow \sqrt{-x^2 - \frac{1}{2}x + 3}$, $\frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} + 4(-1)(-1)(3)}}{-2} = \frac{\frac{1}{2} \pm \sqrt{\frac{49}{4}}}{-2} = \frac{\frac{1}{2} \pm \frac{7}{2}}{-2}$ → $\frac{3}{2}$, -2

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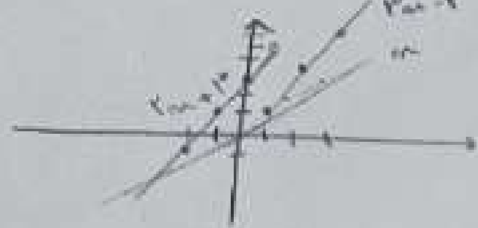
$$f(x)=\begin{cases}3x-2 & x\geq 1\\ 2x+3 & x<1\end{cases}$$



$g(x)=\sqrt{f(x)-x} \Rightarrow f(x)\geq x$

$2x+3=x$

$x=-3$

$(4)\ [-3,+\infty)$

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$$f(x)=\begin{cases}(a+1)(x+2) & x\geq 1\\ 3a+2x & x\leq 1\end{cases}$$

$2f(5)=f(-2)+a \qquad a=?$

$2(a+1)(7)=$

$3a+2(-2)+a$

$14a+14=4a-4$

$10a=-18 \qquad a=-1.8$

$a=-1.8$

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$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$

$f(\underbrace{2-\sqrt{3}}_{t})+f(\underbrace{2+\sqrt{3}}_{\frac{1}{2-\sqrt{3}}})=?$

$\frac{1}{2-\sqrt{3}}\frac{(2+\sqrt{3})}{(2+\sqrt{3})}=\frac{2+\sqrt{3}}{1}$

$f(t)+f(\frac{1}{t})=\sqrt{t}+\frac{1}{\sqrt{t}}+2+\frac{1}{\sqrt{t}}+\frac{1}{\frac{1}{\sqrt{t}}}+2$

$f(t)+f(\frac{1}{t})=2(\sqrt{t}+\frac{1}{\sqrt{t}})+4=2(\underbrace{\sqrt{2+\sqrt{3}}+\frac{1}{\sqrt{2-\sqrt{3}}}}_{S})+4$

$S^2=2-\sqrt{3}+2+\sqrt{3}+2\sqrt{(2-\sqrt{3})(2+\sqrt{3})}$

$4+2\sqrt{1}=6$

$S=\sqrt{6}$

$f(2-\sqrt{3})+f(2+\sqrt{3})=2\sqrt{6}+4$

جواب کلی ۷

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$2f(x)-3f(-x)=4x^2-x \qquad f(x)=?$

$2\times$

$3\times$

$2f(-x)-3f(x)=4x^2-(-x)=4x^2+x$

$4f(x)-6f(-x)=2(4x^2-x)$

$+\ 6f(-x)-9f(x)=3(4x^2+x)$

$-5f(x)=8x^2-2x+12x^2+3x$

$-5f(x)=20x^2+x$

$f(x)=\frac{20x^2+x}{-5}=-4x^2-\frac{x}{5}$

-۹

$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1 \qquad f(0)=?$

$x=-2 \longrightarrow 0-3(-2)f(0)=4(-2)^2-m(-2)+3m-1$

$6f(0)=16+2m+3m-1 \longrightarrow 6f(0)=5m+15$

$x=0 \longrightarrow 2f(0)=3m-1$

$3(3m-1)=5m+15 \rightarrow 9m-3=5m+15$

$4m=18$

$m=\frac{18}{4}=\frac{9}{2}$

$2f(0)=3(\frac{9}{2})-1=\frac{27}{2}-1=\frac{27}{2}-\frac{2}{2}=\frac{25}{2} \rightarrow f(0)=\frac{25}{4}$

-۱۰

$f(x) = ax+b$ $\quad f(x) + f(\frac{1}{x}) = \frac{۳x^۲-۱۲x+۳}{x}$ $\quad f(-1) = ?$

$f(\frac{1}{x}) = \frac{1}{x}a+b$

$ax+b+\frac{a}{x}+b = \frac{۳x^۲-۱۲x+۳}{x}$

$۲b + a(\frac{x^۲+1}{x}) = \frac{۲bx+ax^۲+a}{x} = \frac{۳x^۲-۱۲x+۳}{x}$

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$f(-1) = -a+b \rightarrow -۳+(-۶) = -۹$ $\quad a=۳$ $\quad ۲b=-۱۲$ $\quad b=-۶$

[illegible] ۴ $\quad$ [illegible]