«زهرا الاردانی» «یازدهم (فنی)» «ریاضی ۱» «فصل ۴»

۱-

الف) $f(x)=\sqrt{\dfrac{x-1}{x}-\dfrac{x}{x-1}}$

$D_{f_1}=(-\infty,0)\cup[\frac{1}{2},1)$

$\dfrac{x-1}{x}-\dfrac{x}{x-1}\geq 0$ (جدول تعیین علامت: $0,\ \frac{1}{2},\ 1$: $+\ |\ -\ |\ +\ |\ -$)

$\dfrac{(x-1)^2-x^2}{x^2-x}=\dfrac{x^2-x^2-2x+1}{x^2-x}\rightarrow\dfrac{-2x+1}{x(x-1)}\geq 0$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$ $\rightarrow x\neq 0,\ x\neq -1$ $\rightarrow \dfrac{3}{x-1}+\dfrac{1}{x+2}\neq 0\rightarrow\dfrac{3(x+2)+x-1}{(x-1)(x+2)}\rightarrow\dfrac{4x+5}{(x-1)(x+2)}$

$\rightarrow D_f=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}$

۲-

الف) $y=\sqrt{((\frac{1}{3})^x-9)(32-4^x)}$

$(\frac{1}{3})^x-9=0\rightarrow x=-2$ (جدول: $-2,\ \frac{5}{2}$: $+\ -\ +$)

$32-4^x=0\rightarrow x=\frac{5}{2}$

$D_f=(-\infty,-2]\cup[\frac{5}{2},+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt{y+1}=3-\sqrt[4]{x-1}\Rightarrow 3-\sqrt[4]{x-1}\geq 0$

$\sqrt[4]{x-1}\leq 3\longrightarrow x-1\leq 81$

$x-1\geq 0\rightarrow x\geq 1$

$\rightarrow x\leq 82\rightarrow D_f=[1,82]$

۳-

$f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}\Rightarrow x^2-x-2>0\rightarrow(x-2)(x+1)>0$, $x=2$, $x=-1$ (جدول: $-1,\ 2$: $+\ -\ +$) (۲)

$x^2-1\geq 0\rightarrow x^2\geq 1\rightarrow x\geq 1,\ x\leq -1$ (۱) D_f (۱)$\cap$(۲) $=(-\infty,-1)\cup(2,+\infty)$

۴-

$f(x)=\sqrt{3+ax-x^2}\rightarrow -x^2+ax+3\geq 0$ (جدول: $-2,\ b$: $+\ -\ +$)

$D_f=[-2,b]$ $x^2-ax-3\leq 0$

$-2\times b=-3\rightarrow b=\frac{3}{2}$ $(x+2)(x-b)\leq 0\rightarrow(x+2)(x-\frac{3}{2})\rightarrow x^2+\frac{1}{2}x-3\leq 0$; $a=-\frac{1}{2}$

$a+b=\frac{3}{2}-\frac{1}{2}=1$

۵-

$f(x)=\begin{cases}3x-2, & x\geq 1\\ 2x+3, & x<1\end{cases}\rightarrow 3x-2\geq x\rightarrow x\geq 1$

$g(x)=\sqrt{f(x)-x}$ $2x+3\geq x\rightarrow x\geq -3$

$f(x)-x\geq 0\rightarrow f(x)\geq x$ $D_g=[-3,+\infty)$

۶-

$f(x) = \begin{cases} (a+1)(x+2), & x>1 \\ 3a+2x, & x\leq 1 \end{cases}$

$2f(5) = f(-2) + a$

$a = ?$

$2(a+1)(5+2) = 3a + 2(-2) + a$

$14a + 14 = 4a - 4 \rightarrow 10a = -18$

$a = -1.8$

۷-

$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$

$f(\overbrace{2-\sqrt{3}}^{x_1}) + f(\overbrace{2+\sqrt{3}}^{x_2}) = ?$

معکوس هم هستن چون ضربشان یک میشه

$\sqrt{x_1} + \frac{1}{\sqrt{x_1}} + 2 + \sqrt{x_2} + \frac{1}{\sqrt{x_2}} + 2 = 4 + 2\sqrt{x_1} + 2\sqrt{x_2}$

$= 4 + 2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}} = 4 + 2\sqrt{6}$

$2(\underbrace{\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}}_{x})$

$x^2 = 6 \rightarrow x = \sqrt{6}$

۸-

$2f(x) - 3f(-x) = 4x^2 - x$

$f(x) = ? \rightarrow$ ضرب 2 $\rightarrow 4f(x) - 6f(-x) = 8x^2 - 2x$

ضرب 3 $\rightarrow 6f(-x) - 9f(x) = 12x^2 + 3x$

$-5f(x) = 20x^2 + x \rightarrow f(x) = \frac{20x^2 + x}{-5} = -4x^2 - \frac{x}{5}$

۹-

$(x+2)f(x) - 3x\,f(x+2) = 4x^2 - mx + 3m - 1$

$f(0) = ? \rightarrow x = 0 \rightarrow 2f(0) = 3m - 1$

$f(0) = \frac{3m-1}{2}$

$x = -2 \rightarrow 6f(0) = 16 + 2m + 3m - 1 \rightarrow f(0) = \frac{5m+15}{6}$

$\frac{3m-1}{2} = \frac{5m+15}{6}$

$8m = 36 \rightarrow m = \frac{9}{2}$

$f(0) = \frac{3 \times \frac{9}{2} - 1}{2} = \frac{25}{4}$

۱۰-

$f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x}$

$f(-1) = ? \rightarrow x = -1 \rightarrow 2f(-1) = \frac{3+12+3}{-1} \rightarrow 2f(-1) = -18$

$f(-1) = -9$

$f(x) = ax + b$