نام و نام خانوادگی ......هلیا عطائی...... **پاسخنامه تشریحی تکلیف شماره** ...۴... **کلاس** ...یازدهم دختر C...

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الف) $f(x)=\sqrt{\dfrac{x-1}{x}-\dfrac{x}{x-1}}=\sqrt{\dfrac{(x-1)^2-x^2}{x(x-1)}}=\sqrt{\dfrac{(x-1-x)(x-1+x)}{x(x-1)}}=\sqrt{\dfrac{1-2x}{x(x-1)}}\geq 0$, $x(x-1)\neq 0$

$x \mid 0 \quad \frac{1}{2} \quad 1$: $+ \;\emptyset\; - \;0\; + \;\emptyset\; -$

$D_f:(-\infty,0)\cup(\frac{1}{2},1]$

ب) $\dfrac{\dfrac{1}{x+1}-\dfrac{2}{x}}{\dfrac{3}{x-1}+\dfrac{1}{x+2}}=\dfrac{\dfrac{x-2x-2}{x(x+1)}}{\dfrac{3x+6+x-1}{(x-1)(x+2)}}=\dfrac{\dfrac{-x-2}{x(x+1)\neq 0}}{\dfrac{4x+5}{(x-1)(x+2)\neq 0}\neq 0}$

$$\begin{cases} x(x+1)\neq 0 \rightarrow x\neq 0,\ x\neq -1 \\ 4x+5\neq 0 \rightarrow x\neq \frac{-5}{4} \\ (x-1)(x+2)\neq 0 \rightarrow x\neq 1,\ x\neq -2 \end{cases}$$

$D_f: \mathbb{R}-\{-2,1,-1,0,\frac{-5}{4}\}$

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الف) $f(x)=\sqrt{\left(\left(\frac{1}{3}\right)^x-9\right)\left(32-4^x\right)}\geq 0$

$$\begin{cases} \left(\frac{1}{3}\right)^x=9 \rightarrow \boxed{x=-2} \\ 32=2^{2x} \rightarrow 5=2x \rightarrow \boxed{x=\frac{5}{2}} \end{cases}$$

$-2 \quad \frac{5}{2}$: $+ \;\; - \;\; +$

$D_f:(-\infty,-2]\cup[\frac{5}{2},+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt[4]{x-1}\geq 0 \rightarrow x\geq 1 : \mathrm{I}$

$\sqrt{y+1}=-\sqrt[4]{x-1}+3 \xrightarrow{\sqrt{y+1}\geq 0} -\sqrt[4]{x-1}+3\geq 0$

$\rightarrow -\sqrt[4]{x-1}\geq -3 \rightarrow \sqrt[4]{x-1}\leq 3 \xrightarrow{()^4} x-1\leq 81 \rightarrow x\leq 82 : \mathrm{II}$

$D_f: \mathrm{I}\cap\mathrm{II} : [1,82]$

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$f(x)=\dfrac{\log_4^{x^2-x-2>0}}{\sqrt{x^2-1}+1\neq 0}$

$$\begin{cases} x^2-x-2>0 \\ (x-2)(x+1)>0 \\ \mathrm{I}:(-\infty,-1)\cup(2,+\infty) \end{cases}$$

$x \mid -1 \quad 2$: $+ \;\; - \;\; +$

$$\begin{cases} \sqrt{x^2-1}\geq 0 \\ (x-1)(x+1)\geq 0 \\ \mathrm{II}:(-\infty,-1]\cup[1,+\infty) \end{cases}$$

$-1 \quad +1$: $+ \;\; - \;\; +$

$\{\sqrt{x^2-1}\neq -1$ ✓

$D_f: \mathrm{I}\cap\mathrm{II} : (-\infty,-1)\cup(2,+\infty)$

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$\sqrt{-x^2+ax+3}\geq 0 \qquad D_f:[-2,b] \qquad a+b=\frac{-1}{2}+\frac{3}{2}=1$

$\xrightarrow{x=-2} -4-2a+3=0 \rightarrow \boxed{a=\frac{-1}{2}}$

$\rightarrow -x^2-\frac{1}{2}x+3\geq 0 \xrightarrow{\Delta=b^2-4ac} \Delta=\frac{1}{4}\overbrace{-4(-1)(3)}^{+12}=\frac{49}{4}$

$x=\dfrac{-b\pm\sqrt{\Delta}}{2a} \longrightarrow x=\dfrac{\frac{1}{2}\pm\frac{7}{2}}{-2}=\begin{cases} x_1=\dfrac{\frac{8}{2}}{-2}=-2 \\ x_2=\dfrac{\frac{-6}{2}}{-2}=\frac{3}{2} \end{cases} \longrightarrow \boxed{b=x_2=\frac{3}{2}}$

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روش ۱:

$f(x)+f\left(\frac{1}{x}\right)=\dfrac{3x^2-12x+3}{x} \xrightarrow{x=-1} 2f(-1)=\dfrac{3+12+3}{-1}=-18 \rightarrow f(-1)=-9$

روش ۲:

$f(x)=ax+b$

$f\left(\frac{1}{x}\right)=\frac{a}{x}+b$

$\rightarrow ax+\frac{a}{x}+2b=3x-12+\frac{3}{x} \Rightarrow \begin{cases} a=3 \\ b=-6 \end{cases}$

$f(x)=3x-6$

$f(-1)=-3-6=-9$

$f(x) = \begin{cases} (a+1)(x+2) & ; x \geq 1 \\ 3a+2x & ; x < 1 \end{cases}$ ۶

$2f(5) = f(-2) + a$

$f(5) = (a+1)(7) = 7a+7 \xrightarrow{\times 2} 2f(5) = 14a+14$

$f(-2) = 3a-4 \xrightarrow{+a} 4a-4$

$\rightarrow 14a+14 = 4a-4$

$7a+7 = 2a-2$

$5a = -9$

$a = \frac{-9}{5}$

$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$

$f(2-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1(\sqrt{2+\sqrt{3}})}{\sqrt{2-\sqrt{3}} \times (\sqrt{2+\sqrt{3}})} + 2 = \overbrace{\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}}^{A} + 2$

$f(2+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 = \overbrace{\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}}^{A} + 2$

$f(2-\sqrt{3}) + f(2+\sqrt{3}) = 2A + 4 = 2\sqrt{6} + 4$

$A^2 = 2 - \cancel{\sqrt{3}} + 2 + \cancel{\sqrt{3}} + 2$

$A = 6$ ق ق

$A = \pm\sqrt{6} \rightarrow A = \sqrt{6}$

غ ق ق (A جمع دو رادیکال است، نامنفی) ۷

$2f(x) - 3f(-x) = 4x^2 - x \xrightarrow{\times 2} 4f(x) - 6f(-x) = 8x^2 - 2x$

$\xrightarrow{x=-x} 2f(-x) - 3f(x) = 4x^2 + x \xrightarrow{\times 3} 6f(-x) - 9f(x) = 12x^2 + 3x$

$\Rightarrow -5f(x) = 20x^2 + x$

$\rightarrow f(x) = -4x^2 - \frac{x}{5}$

$f(x) = -4x^2 - \frac{x}{5}$ ۸

$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$

$\xrightarrow{x=0} 2f(0) = 3m-1 \xrightarrow{\times 3} 6f(0) = 9m-3$

$\xrightarrow{x=-2} 6f(0) = 16 + 2m + 3m - 1 = 15 + 5m$

$\xrightarrow{6f(0) = 6f(0)} 9m - 3 = 15 + 5m$

$4m = 18 \rightarrow m = \frac{9}{2}$ ۹

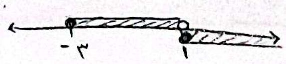
$f(0) = \frac{3m-1}{2} \xrightarrow{m = \frac{9}{2}} f(0) = \frac{\frac{27}{2} - \frac{2}{2}}{2} = \frac{25}{4}$

$g(x) = \sqrt{f(x) - x}$

$f(x) = \begin{cases} 3x-2 & ; x \geq 1 \\ 2x+3 & ; x < 1 \end{cases}$

$2x+3-x = x+3$	1	$3x-2-x = 2x-2$
	$3-2-1 = 0$	

$\begin{cases} x+3 \geq 0 \rightarrow x \geq -3 \xrightarrow{x<1} -3 \leq x < 1 : \mathrm{I} \\ 2x-2 \geq 0 \rightarrow x \geq 1 \xrightarrow{x \geq 1} x \geq 1 : \mathrm{II} \\ 0 \geq 0 \checkmark \end{cases}$ ۵



باشد $D_g : \mathrm{I} \cup \mathrm{II} : [-3, +\infty)$